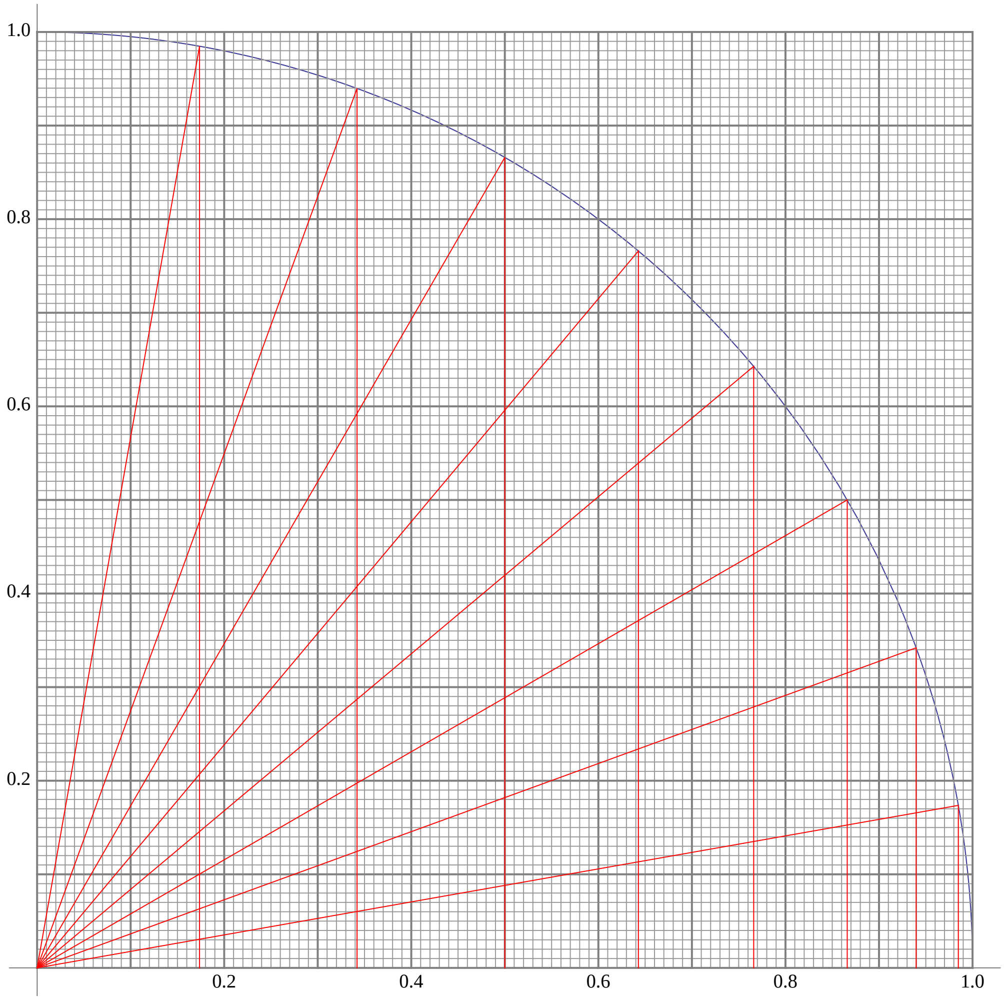
Using Right Triangles to Tabulate Output Values of Sine and Cosine

The diagram below shows right triangles in standard position with hypotenueses of unit length. These triangles are drawn on a cooridinate grid with fine lines spaced at $\frac{1}{100}$ of a unit. The angles shown at the origin of the coordinate grid are multiples of 10 °. For these input angles θ, we can tabulate the values of cos(θ) to within a two decimal places (perhaps three) by carefully reading off the lengths of the horizontal legs. Similarly, we can tabulate the values of sin(θ), by carefully reading off the lengths of the vertical legs. These values appear in the table below the diagram.



θ	10 °	20 °	30 °	40 °	50 °	60 °	70 °	80 °
cos(θ)	0.985	0.940	0.866	0.766	0.643	0.500	0.342	0.174
sin(θ)	0.174	0.342	0.500	0.643	0.766	0.866	0.940	0.985

Make sure you understand the relationship here...check for yourself that the vertical leg lengths correspond to the entries for sine above, and the horizontal leg lengths correspond to the values for cosine. Next, note that as the angle θ increases from 0° to 90°, the values of the sines of those angles also increase from 0 to 1. This implies the sine function is a "1-to-1" function, and thus it has an inverse function (called "the **inverse sine function**"). That is, given any number, 0 < y < 1, there is exactly one angle 0 ° < θ < 90 ° having that number y as its sine value. We write $\theta = \sin^{-1}(y)$.

Similarly, as the angle θ increases from 0° to 90°, the values of the cosines of those angles decrease from 1 to 0. This implies the cosine function is a "1-to-1" function as well, and thus has an inverse function, which we call "the **inverse cosine function**". In particular, given any number 1 > x > 0, there is exactly one angle 0 °< θ < 90 ° having that number x as its cosine value. In this case, we right $\theta = \cos^{-1}(x)$.

Exercise 1: From the table above, find the values of : **a)** $\sin^{-1}(.500)$ **b)** $\sin^{-1}(.940)$ **c)** $\cos^{-1}(.766)$ **d)** $\cos^{-1}(.342)$

Exercise 2: Using the graph and a protractor to measure the approximate values of : **a)** $\sin^{-1}(.80)$ **b)** $\sin^{-1}(.30)$ **c)** $\cos^{-1}(.80)$ **d)** $\cos^{-1}(.30)$