Basic Properties of the Three Fundamental Set Operations: 
Union, Intersection, and Complement

For any subsets \( A, B, C \) of a given universal set \( U \), we have:

1. \( \phi \cup A = A \)  \hspace{1cm} 2. \( \phi \cap A = \phi \)  \hspace{1cm} 3. \( U \cup A = U \)  \hspace{1cm} 4. \( U \cap A = A \)  \hspace{1cm} (Special Subsets)
5. \( A \cup A = A \)  \hspace{1cm} 6. \( A \cap A = A \)
7. \( A \cup A' = U \)  \hspace{1cm} 8. \( A \cap A' = \phi \)  \hspace{1cm} 9. \( (A')' = A \)
10. \( A \cup B = B \cup A \)  \hspace{1cm} 11. \( A \cap B = B \cap A \)  \hspace{1cm} (Commutative Properties)
12. \( (A \cup B) \cup C = A \cup (B \cup C) \)  \hspace{1cm} 13. \( (A \cap B) \cap C = A \cap (B \cap C) \)  \hspace{1cm} (Associative Properties)
14. \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)  \hspace{1cm} (Intersection Distributes over Union)
15. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)  \hspace{1cm} (Union Distributes over Intersection!!)
16. \( (A \cup B)' = A' \cap B' \)  \hspace{1cm} 17. \( (A \cap B)' = A' \cup B' \)  \hspace{1cm} (DeMorgan’s Laws)

Exercise 1a: Use property 12 to show that: \((A \cup B) \cup (D \cup E) = A \cup (B \cup (D \cup E)) \)  [Hint: Let \( C = D \cup E \)]
Exercise 1b: Use property 12 to show that: \((A \cup (D \cup E)) \cup F = A \cup (D \cup (E \cup F)) \)  [Hint: Let \( B = D \cup E \)]

Note: Associativity of union implies that parentheses aren’t needed when writing the union of any number of sets.
Similarly, associativity of intersection implies parentheses aren’t needed when intersecting any number of sets.
...but parentheses are needed when a mixture of unions and intersections are present!

Exercise 2: Show that \((A \cup C \cup D)' = A' \cap C' \cap D' \)  [Hint: Use property 16 twice, first with \( B = C \cup D \)]
Exercise 3: There is a rough analogy between the algebra of the integers \( \mathbb{Z} \) together with the operations addition, multiplication, and “taking the opposite”...and the algebra of all subsets of \( U \) (i.e. \( \mathcal{P}(U) \)) together with the operations of union, intersection, and “taking the complement”. Taking the correspondence: \( \cup \leftrightarrow +, \cap \leftrightarrow \cdot, \text{and} \rightarrow \leftrightarrow ' \), which of properties analogous to 1-17 above hold for arbitrary numbers \( a, b, c \in \mathbb{Z} \). Which of the analogous properties do not hold?

Further Exploration of Venn Diagrams and Set Operations

- Introduction

Venn Diagrams are used to help visualize relationships among sets. The conventional Venn Diagram consists of an outer bounding rectangle (representing the “universal set” in the context of the given discussion), and a collection of simple closed curves...one such curve for each set under consideration. Typically circles are used to depict the sets if there are 3 or fewer sets, otherwise other simple, closed curves are used.

(Note: A curve is closed if it begins and ends at the same point. A curve is simple if it does not intersect itself.)

In the two examples below, we introduce the idea of an “indivisible set” in a Venn Diagram...

Example 1: For a Venn Diagram with two sets \( A_1 \) and \( A_2 \) the indivisible subsets are:

\[
A_1 \cap A_2, \ A_1 \cap A_2', \ A_1' \cap A_2, \ \text{and} \ A_1' \cap A_2'
\]

Shade each of these sets in a different color in the Venn Diagram.

Example 2: For a Venn Diagram with three sets \( A_1, \ A_2, \ \text{and} \ A_3 \) the indivisible subsets are:

\[
A_1 \cap A_2 \cap A_3, \ A_1 \cap A_2 \cap A_3', \ A_1 \cap A_2' \cap A_3, \ A_1 \cap A_2' \cap A_3'
\]

\[
A_1' \cap A_2 \cap A_3, \ A_1' \cap A_2 \cap A_3', \ A_1' \cap A_2' \cap A_3, \ A_1' \cap A_2' \cap A_3'
\]