City College of San Francisco **ASTR 20:** Introduction to Astrophysics
Spring 2010, Assignment # 1

Due Date: February 11, 2010

<u>Reading Assignment:</u>

Jones & Lambourne: Chapter 1: The Milky Way

<u>Problems:</u>

1. The mean densities of stars can vary by enormous factors. For purposes of illustration, calculate the mean densities for each of the following:

   a). the Sun,
   b). the supergiant star Betelgeuse, with a mass of 10 $\mathfrak{M}_\odot$ and a radius of 300 $\mathcal{R}_\odot$,
   c). a 1.4 $\mathfrak{M}_\odot$ white dwarf, with a radius of 5 x 10^7 m, and
   d). a 1.4 $\mathfrak{M}_\odot$ neutron star, with a radius of 2 x 10^4 m.

2. Mass Profile $M(r)$, Density Profile $\rho(r)$, and Velocity Profile $v(r)$:

   Use the Virial Mass Estimate formula derived in class [Equation 1.4 & 1.5] to derive density profile and velocity profile of the system for the following cases:

   a). $M(r)$ = Constant.= M, indicating as a mass point. In this case, further show that the velocity profile implies Kepler's 3rd Law,
   b). $M(r)$ decreases as $1/r$,
   c). $M(r)$ increases linearly with r,
   d). $M(r)$ increases as r^2, and
   e). $M(r)$ increases as r^3

3. Apply the Virial Theorem to derive the **Virial Mass** M in **solar mass unit** enclosed in a volume with radius r in **kpc unit** with a test particle with mass m and orbital velocity of v in **km/s unit**. Apply the Virial Mass formula and estimate the mass of the Milky Way galaxy in solar masses at a distance of 15 kpc from the center where a faint star with orbital velocity of 230 km/s is detected. [Hint: Gravitational Binding Energy, page 106.]

4. You are to make use of observational data in order to make a table of the luminosities, surface temperatures, radii, and mean densities of main-sequence stars of 50, 1.0, and 0.1 solar masses. Show your calculations, or, at least, examples of your calculations.

   a). With the aid of the mass-luminosity relation $(L / L_\odot) = (M / M_\odot)^3$ and the Hertzsprung-Russell diagram, tabulate the luminosities (in units of the solar luminosity) and surface temperatures of stars of 50, 1.0, and 0.1 solar masses.
   b). With the aid of Stefan-Boltzmann law, $L = 4\pi R^2 \sigma T^4$, determine the radii (in units of solar radius) of stars of 50, 1.0, 0.1 solar masses.
   c). Calculate the mean densities of stars of the three masses in kg / m^3.

d). In a simple sentence, describe the qualitative relationship between the mass and mean density of a star that is indicated by your results. (In other words, what is the trend in the variation of the mean densities of stars with their masses?

Optional Bonus Problem:

This problem is concerned with the simple derivation of Kepler's Law:

$$m_1 + m_2 = \frac{4\pi^2}{G}\frac{R^3}{P^2}$$

a). Draw a picture of two stars rotating around the center of mass, so $m_1 r_1 = m_2 r_2$.
b). Equate the central force on star 1 to the gravitational force between the stars. The force acts over a distance of $R = r_1 + r_2$.
c). Noting that both stars have the same orbital period, $P = P_1 = P_2 = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$, solve for v_1 in terms of P and r_1 . Substitute for v_1 in part (b).
d). Note that $R = r_1(1 + \frac{r_2}{r_1})$.
e). Now solve for r_2 / r_1 from the center of mass definition, in terms of m_1 and m_2 , and substitute for r_2 / r_1 in part (d) with the relevant mass ratio. Solve for r_1 in terms of R, m_1, and m_2.
f). Group constants and R on one side, and terms in m and P on the other. Note that $m_1 + m_2 \equiv \mathfrak{M}$.