

Project Note # 3: Properties of Telescope

A. Light-Gathering Power and Limiting Magnitudes:

The **light-gathering power** of a telescope is a measure of how much light (photons) that it can gather as compared to another one of identical design but with different aperture size. Thus, the light-gathering power of a telescope is directly proportional to its aperture diameter-squared (i.e. area of the primary mirror for reflector). For example, a 12-inch reflector telescope has a light-gathering power about 2500X greater than our dilated eyes ($d = 6$ mm). Therefore, if we use a 12-inch reflector we can observe objects that are 2500X fainter than with our naked eyes! As we know the faintest star we can see with our naked eyes has a visual apparent magnitude $V = 6.0$. What would be the visual apparent magnitude of the faintest star that we could see with the aid of the 12-inch reflector? We could estimate V by using the definition of magnitude and the following formula:

$$m_1 - m_2 = -2.5 \log_{10} [b_1 / b_2]$$

where m 's are magnitudes, and b 's are the brightness respectively. Let m_1 be the visual apparent magnitude of the faintest star we could observe with the aid of the 12-inch reflector and b_1 the corresponding brightness; whereas m_2 and b_2 are the magnitude and brightness without the telescope. Then, $m_1 = m_1$ (eye) , $m_2 = 6.0$, $b_1 = [1/2500] b_2$,

$$m_1 \text{ (eye)} - 6.0 = -2.5 \log_{10} [1/2500] .$$

The **limiting magnitude**, m_1 (eye) = 14.5 . The limiting magnitude of an optical system is the faintest magnitude that the optical system can detect. (i.e. The limiting magnitude for our eyes in the visual band is 6.0, whereas the combination of the 12-inch reflector and our eyes could have a limiting magnitude of 14.5.). If we use other kinds of detectors other than our eyes with the 12-inch reflector, we will get another value for the limiting magnitude for the system. In fact, our eyes are not very efficient in counting photons. How about photographic plates and films? They are also not efficient in counting photons. However, we can expose the photographic materials with long exposure time to achieve fainter magnitudes, whereas our eyes could not! The concept of limiting magnitude is meaningful only when we know what kind of detector we are using and what is the typical exposure time of integration. Otherwise, it is difficult to compare one with the other.

CCD cameras are becoming more accessible to both professional and amateur astronomers, and they are very efficient photon counters. It is typical at least > 100X more efficient than our eyes and photographic materials. Thus, if we use a CCD camera and estimate the limiting magnitude as above with the use of the 12-inches reflector, $m_l(\text{CCD}) \geq 19.5$ (remember 100X difference in brightness corresponds to 5.0 mag. difference).

Once we know the limiting magnitude of the telescope-detector system, we can estimate how far we can detect a supernova through the use of the distance modulus formula. For our eyes, we can see a Type Ia supernova to a distance of 1 Mpc (assume $M_V = -19$); whereas the 12-inches reflector with a CCD camera can detect a Type Ia supernova to few hundred megaparsecs away.

B. Angular Resolution:

The **angular resolution** of a telescope is the smallest angular separation of two point sources that the telescope is able to separate. High **angular resolving power** means the ability to resolve small angular separation. The angular resolution of a telescope operating in the optical region could be estimated by the following formula:

$$\theta'' \text{ (arcsecond)} = 0''.25 \left[\lambda (\mu) / d \text{ (m)} \right]$$

where the angular resolution in arcseconds, the observed wavelength $\lambda (\mu)$ in microns ($1\mu = 10^{-6} \text{ m}$), and $d \text{ (m)}$ is the diameter of the primary mirror of the telescope in meters. For example, our dilated eyes ($d = 6 \text{ mm}$) have an angular resolution of about 20 arcseconds, whereas a 12-inches telescope has an angular resolution of 0.4 arcsecond for $\lambda (\mu) = 0.5\mu$.

However, ground-based observations are limited by the atmosphere. The atmosphere blurs a point source image to an apparent angular size of few arcseconds. This phenomenon is called **seeing**. Typical seeing is about one to a few arcseconds, and only in some pristine sites will we occasionally have seeing of 0.3 arcsecond. We would like to get a telescope that has angular resolution smaller than the seeing. (Why?)

C. Field-Of-View (FOV)

The **field-of-view** (FOV) of a telescope is a measurement of the projected angular size of the sky imaged onto the detector.

$$\text{FOV} = \left[206,265'' / F \right] \times L$$

where F is the **focal length** of the telescope and L is the linear dimension of the detector. How do we know the focal length of the telescope? The focal length could be calculated by multiplying the **focal ratio** ($f/\#$) to the aperture diameter of the telescope. For example, a 12-inch telescope with $f/10$, then the focal length $F = (12 \text{ inches}) \times 10 = 120 \text{ inches}$. Thus, if we use a square CCD camera (the typical size of a detector is $1 \text{ cm} \times 1 \text{ cm}$), then the $\text{FOV} = 11'.3 \times 11'.3$ ($1 \text{ arcminute} = 1'.0 = 60 \text{ arcsecond} = 60''.0$).
