Project Note # 4: A Detection Model

The following is an example of calculating the exposure time required to detect a supernova with V < 18 with a 12-inch Schmidt-Cassegrain f/10 telescope and using a CCD camera (800 x 800 pixel-squared). This is called a **detection model**.

The quantitative description of the criteria for detection is the estimation of the **signal-to-noise ratio** (<SNR>) of the signal (i.e. the total amount of photons from the supernova as compare to the background photons from the sky). A <SNR> of 3 - 5 will usually constitute a detection.

The following is a “standard” model to estimate the signal <S>:

$$<S> \quad = F_\lambda \, \Delta\lambda \; T \; A \; \varepsilon \; q \; t_{exp} \quad \text{[photons]},$$

where

F_λ = flux of the supernova in photons / cm^2 / sec / Angstrom
$\Delta\lambda$ = bandwidth of the filter in Angstrom (=880 Angstrom for V band)
T = atmosphere transmission (between 0 to 1, 0 = no transmission and 1 = complete transmission. In the V-band window it is about 0.7)
A = effective collecting area of the primary mirror in cm^2 (e.g. a 12-inch Schmidt-Cassegrain telescope will have an effective area of $0.9 \times \pi (6 \times 2.54)^2$ cm^2 = 657 cm^2)
ε = efficiency of the optical system (each optical element, such as a mirror is about 90% efficient in reflecting photons, typically, we will have 5 elements => 0.9 x 0.9 x 0.9 x 0.9 x 0.9 = 0.6)
q = quantum efficiency of the CCD camera (0.5 in V-band)
t_{exp} = exposure time in seconds

and the noise <N>:

$$<N> \quad = [\; <S> + 2<B> \;\}^{1/2} \qquad \text{, where } <B> = \text{background photons from sky}$$

$$<B> \quad = f_\lambda \, \theta^2 \Delta\lambda \; T \; A \; \varepsilon \; q \; t_{exp}$$

f_λ = surface brightness of the sky in photons / cm^2 / sec / Angstrom / arcsecond^2
θ^2 = angular area of sky that contributes to the background in arcsecond^2

<S> and <B> reduce to:

$$<S> \quad = 121413.6 \; F_\lambda t_{exp} \qquad \text{and} \qquad <B> \quad = 121413.6 \; f_\lambda \, \theta^2 t_{exp} \; .$$

Next, we will estimate the flux from a supernova with V = 18 and the surface brightness of the sky. An object with V = 0 has a flux of 1000 photons / cm^2 / sec / Angstrom. Thus, V = 18 will have a flux of 6.31 x 10^{-5} photons / cm^2 / sec / Angstrom (using the relationship between magnitude and flux). Therefore, <S> = 7.66 t_{exp} [photons].

We then proceed to estimate the flux due to the night sky. In Chicago, the surface brightness of night sky is about 18 mag/ $arcsecond^2$; this is equal to 6.31 x 10^{-5} photons / cm^2 / sec / Angstrom / $arcsecond^2$. The flux due to the night sky is the product of the surface brightness of the night sky and the angular area ($f_\lambda \theta^2$). Since we are measuring point source the angular area from the sky that contributes to noise is comparable to the seeing , about 2 arcsecond. Thus, the angular area is about 3.2 $arcsecond^2$. Therefore, the flux due to night sky is 2.03 x 10^{-4} photons / cm^2 / sec / Angstrom, and <B> = 24.5 t_{exp} [photons] .

Finally, we get the <SNR> :

$$\langle SNR \rangle = \langle S \rangle / \langle N \rangle = 1.02 \, [\, t_{exp} \,]^{1/2} ,$$

Thus, for detection level set at <SNR> = 5, we need about 25 seconds of integration time with the CCD camera. However, if we want a reasonable photometry for the supernova, we need at least a <SNR> > 25. This implies we need to have at least 10 minutes of integration time.