**Set #1 – Rectilinear Motion**

**1.** The motion of a particle is defined by the relation $x = 3t^4 + 4t^3 - 7t^2 - 5t + 8$, where x and t are expressed in millimeters and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when t = 3s.

Given: $x = 3t^4 + 4t^3 - 7t^2 - 5t + 8$

Find: $x|_{t=3}$

$v|_{t=3}$

$a|_{t=3}$

Solution:

We need $x(t)$, $v(t)$, $a(t)$.

We are given $x(t)$:

$$x(3) = 3(3)^4 + 4(3)^3 - 7(3)^2 - 5(3) + 8$$
$$= \underline{\underline{281\ \text{mm}}}$$

To get $v(t)$, use $v = \frac{dx}{dt}$

$$v = 12t^3 + 12t^2 - 14t - 5$$
$$v(3) = 12(3)^3 + 12(3)^2 - 14(3) - 5$$
$$= \underline{\underline{385\ \text{mm/s}}}$$

To get $a(t)$, use $a = \frac{dv}{dt}$

$$a = 36t^2 + 24t - 14$$
$$a(3) = 36(3)^2 + 24(3) - 14$$
$$= \underline{\underline{382\ \text{mm/s}^2}}$$

**Set #1 – Rectilinear Motion**

**2.** The acceleration of a particle is directly proportional to the time t. At t = 0, the velocity of the particle is v = 16 inches/s. Knowing that v = 15 inches/s and that x = 20 inches when t = 1 s, determine the velocity, the position, and the total distance traveled when t = 7 s.

Given: $a = kt$

$$\left.\begin{aligned} v|_{t=0} &= 16 \text{ in/s} \\ v|_{t=1s} &= 15 \text{ in/s} \\ x|_{t=1s} &= 20 \text{ in} \end{aligned}\right\} \text{Boundary Conditions (B.C.)}$$

Find: $v|_{t=7s}$

$x|_{t=7s}$

distance traveled$|_{t=7s}$

Solution:

Note: position does not necessarily equal the distance traveled.

position $\neq$ distance traveled

To find $v|_{t=7s}$, we need $v(t)$. Use $a = \frac{dv}{dt}$

$\frac{dv}{dt} = kt$ & use $1^{st}$ B.C.

$$\longrightarrow \int_{16}^{v} dv = \int_{0}^{t} kt\, dt$$

$$v - 16 = \frac{kt^2}{2}$$

$$v = 16 + \frac{kt^2}{2}$$

Now, use $2^{nd}$ B.C.

Set #1

2. continued.

Now, use $2^{nd}$ B.C.

$$15 = 16 + \frac{k(1)^2}{2} \longrightarrow k = -2$$ [Note: units for k are in/s$^2$]

$$\longrightarrow v(t) = 16 + \frac{(\cancel{-2})\, t^2}{\cancel{2}}$$

$$v(t) = 16 - \frac{t^2}{2}$$

$$v(7\text{ sec}) = 16 - (7)^2$$

$$= \underline{\underline{-33\ \text{in}/\text{s}}}$$

To find $x|_{t=7}$, we need $x(t)$. Use $v = \frac{dx}{dt}$

$$\frac{dx}{dt} = 16 - t^2 \quad \text{\& use } 3^{rd} \text{ B.C.}$$

$$\longrightarrow \int_{20}^{x} dx = \int_{1}^{t} (16 - t^2)\, dt$$

$$x - 20 = \left[16t - \frac{t^3}{3}\right]_1^t$$

$$x = 20 + \left(16t - \frac{t^3}{3}\right) - \left(16 - \frac{1}{3}\right)$$

$$x(t) = -0.33t^3 + 16t + 4.33$$

$$x(7) = -0.33(7)^3 + 16(7) + 4.33$$

$$= \underline{\underline{2\ \text{in}}}$$

To find total distance traveled, we must consider if the particle changed direction, i.e. is $v=0$ at any time between 0 sec and 7 sec.

$$v(t) = 16 - t^2$$

$$0 = 16 - t^2$$

$$t^2 = 16$$

$$t = \pm 4 \longrightarrow \therefore \text{ Particle turns around at } t = 4 \text{ sec!}$$

Set #1

2. continued

So total distance traveled is:

$$|x(4) - x(0)| + |x(7) - x(4)|$$

$$x(7) = -0.33(7)^3 + 16(7) + 4.33$$
$$= 2 \text{ in}$$
$$x(4) = -0.33(4)^3 + 16(4) + 4.33$$
$$= 47 \text{ in}$$
$$x(0) = 4.33 \text{ in}$$

Total distance traveled =

$$= |47 - 4.33| + |2 - 47|$$
$$= \underline{\underline{87.67 \text{ in}}}$$

**Set #1 – Rectilinear Motion**

**3.** The acceleration of a particle is defined by the relation a = $-k\sqrt{v}$, where k is a constant. Knowing that x = 0 and v = 81 m/s at t = 0 and that v = 36 m/s when x = 18 m, determine

a) the velocity of the particle when x = 20 m,

b) the time required for the particle to come to rest.

Given: $a = -k\sqrt{v}$ (k is constant)

$$\left.\begin{array}{l} v|_{t=0} = 81 \text{ m/s} \\ x|_{t=0} = 0\text{m} \\ v|_{x=18\text{m}} = 36 \text{ m/s} \end{array}\right\}$$ These are called Boundary Conditions (B.C.)

Find: (a) $v|_{x=20\text{m}}$

(b) t when particle comes to rest.

Solution:

(a) To find v at x = 20 m, we need to have v(x)

→ use $a = v\dfrac{dv}{dx}$

$$-k\sqrt{v} = v\frac{dv}{dx}$$

$$-k\,dx = v^{1/2}\,dv$$

Using the first two boundary conditions, we can put limits on the integrals:

$$-\int_0^x k\,dx = \int_{81}^v v^{1/2}\,dx$$

$$-kx = \left[\frac{v^{3/2}}{3/2}\right]_{81}^{v}$$

$$= \frac{v^{3/2}}{3/2} - \frac{81^{3/2}}{3/2}$$

$$= 2/3\, v^{3/2} - 486$$

Solving for v:

→ $2/3\, v^{3/2} = 486 - kx$

$v^{3/2} = 729 - 3/2\, kx$ —(1)

Using last B.C.:

→ $36^{3/2} = 729 - 3/2\, k(18)$

$k = 19$

Set #1

3. continued

Substituting the value for k into ①.

$\rightarrow \quad v^{3/2} = 729 - \frac{3}{2} k x$

$v^{3/2} = 729 - \frac{3}{2}(19)\, x$

$v^{3/2} = 729 - 28.5\, x \quad — ②$

Solving ② for v in terms of x

$\rightarrow \quad v(x) = [729 - 28.5\, x]^{2/3}$

$v(20) = [729 - 28.5\,(20)]^{2/3}$

$= \underline{\underline{29.3 \text{ m/s}}}$

(b.) Now, we need v(t)!

$\rightarrow$ Use $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = -k\sqrt{v}$$

$$\frac{dv}{\sqrt{v}} = -k\,dt$$

Using first B.C., we can set limits on integrals:

$$\int_{81}^{v} v^{-1/2}\, dv = -k \int_{0}^{t} dt$$

$$\left[\frac{v^{1/2}}{1/2}\right]_{81}^{v} = -kt$$

$2v^{1/2} - 2(9) = -kt$

$2v^{1/2} - 18 = -kt$

$v^{1/2} = \frac{1}{2}(18 - kt) \quad \leftarrow k = 19$ from before

$= \frac{1}{2}(18 - 19t)$

$= 9 - 9.5t$

$$v(t) = (9 - 9.5t)^2$$

To find when v=0, solve for t:

$\rightarrow \quad 0 = (9 - 9.5t)^2$

$t = \frac{9}{9.5}$

$= \underline{\underline{0.947 \text{ sec}}}$

**Set #1 – Rectilinear Motion**

**4.** A particle is projected to the right from the position x = 0 with an initial velocity of 9 m/s. If the acceleration of the particle is defined by the relation $a = -0.6v^{3/2}$, where a and v are expressed in $m/s^2$ and m/s, respectively, determine

a) the distance the particle will have traveled when its velocity is 4 m/s,
b) the time when v = 1 m/s,
c) the time required for the particle to travel 6 m.

Given: $a = -0.6\,v^{3/2}$ (units in m & s)

$$\left.\begin{array}{l} x\big|_{t=0} = 0\ m/s \\ v\big|_{t=0} = 9\ m/s \end{array}\right\}$$ Boundary Conditions (B.C.)

Find: (a) distance travelled when $v = 4\ m/s$

(b) $t\big|_{v=1\ m/s}$

(c) time required for particle to travel 6m

Solution:

Note: position does not necessarily equal the distance traveled, in general.

position ≠ distance traveled.

Note: a is defined only for positive v, so, in fact, the particle only has positive velocity. (It never turns around.) Hence, for this Problem,

position = distance traveled

(a.) We need v(x). Use $a = v\dfrac{dv}{dx}$

$$v\frac{dv}{dt} = -0.6\,v^{3/2}$$

Using 1st B.C.

$$\int_9^v v^{-1/2}\,dv = -0.6\int_0^x dx$$

$$\left[2v^{1/2}\right]_9^v = -0.6\,x$$

$$2v^{1/2} - 6 = -0.6x$$

Set #1

4. continued

$$\int_9^v v^{-1/2}\,dv = -0.6\int_0^x dx$$

$$\left[2v^{1/2}\right]_9^v = -0.6x$$

$$2v^{1/2} - 6 = -0.6x$$
$$v^{1/2} = 3 - 0.3x$$
$$3.33\,v^{1/2} = 10 - x$$

$$x(v) = 10 - 3.33\,v^{1/2}$$
$$x(4) = 10 - 3.33\,(4)^{1/2}$$
$$= \underline{\underline{3.33\text{ m}}}$$

(b.) We need $v(t)$. Use $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = -0.6\,v^{3/2}$$
$$\int_9^v v^{-3/2}\,dv = -0.6\int_0^t dt$$

$$\left[-2v^{-1/2}\right]_9^v = -0.6t$$

$$v^{-1/2} - 9^{-1/2} = 0.3t$$
$$v^{-1/2} = 0.33 + 0.3t$$

$$(1\,\text{m/s})^{-1/2} = 0.33 + 0.3t$$
$$\left(1 - \tfrac{1}{3}\right)\frac{1}{0.3} = t$$

$$\underline{\underline{t = 2.22\text{ sec}}}$$

(c.) We need $x(t)$. We could get it, but, to avoid extra work, let's use what we have. Let's find $v$ when $x = 6$m so we can use this in $v(t)$ to find $t$.

$$v^{1/2}(x) = 3 - 0.3x$$
$$v(6\text{m}) = (3 - 0.3(6))^2$$
$$= 1.44\text{ m/s}$$

$$v^{-1/2} = 0.33 + 0.3t$$
$$(1.44)^{-1/2} = 0.33 + 0.3t$$
$$\underline{\underline{t = 1.67\text{ s}}}$$

Set #1

4. continued

(c.) Alternative - Let's find $x(t)$

We have $v(x)$ and $v(t)$. Use $v(t) = \frac{dx}{dt}$

$$v(t) = (0.33 + 0.3t)^{-2} = \frac{dx}{dt}$$

$$dx = \frac{dt}{(0.33 + 0.3t)^2}$$

Let $u = (0.33 + 0.3t)$ $\rightarrow$ at $t = 0$, $u = 0.33$ & $x = 0$

$du = 0.3\,dt$

$$dx = \frac{du}{0.3\,u^2}$$

$$0.3 \int_0^x dx = \int_{0.33}^{u} u^{-2}\,du$$

$$0.3\,x = \left[-u^{-1}\right]_{0.33}^{u}$$

$$0.3\,x - 3 = \frac{-1}{u} + \frac{1}{0.33}$$

$$0.3x - 3 = \frac{-1}{u}$$

$$u = \frac{1}{3 - 0.3\,x}$$

$$0.33 + 0.3t = \frac{1}{3 - 0.3x}$$

when $x = 6$, $0.33 + 0.3t = \frac{1}{3 - 1.8}$

$$0.33 + 0.3t = 0.833$$

$$\underline{\underline{t = 1.67 \text{ sec}}}$$