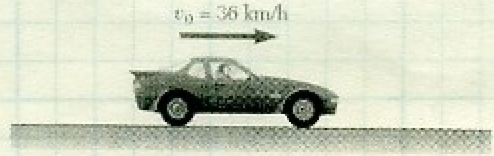
Set #2 – Uniformly Accelerated Motion

1. A motorist enters a freeway at 36 km/h and accelerates uniformly to 90 km/h. From the odometer in the car, the motorist knows that she traveled 0.2 km while accelerating. Determine

a) the acceleration of the car,

b) the time required to reach 90 km/h.



Given: $v_o = 36 \text{ km/hr}$

a = constant

$v_f = 90 \text{ km/hr}$

$d = x_f - x_o = 0.2 \text{ km}$

Find: (a) a

(b) t required to reach 90 km

Solution: Constant Acceleration → Uniformly Accelerated Rectilinear Motion

(a) We can use $v_f^2 = v_o^2 + 2a(x_f - x_o)$

$$90^2 = 36^2 + 2a(0.2)$$

$$a = 17010 \text{ km/hr}^2$$

$$\underline{\underline{a = 1.3125 \text{ m/s}^2}}$$

(b.) Use $v_f = v_o + at$

$$90 = 36 + (17000)t$$

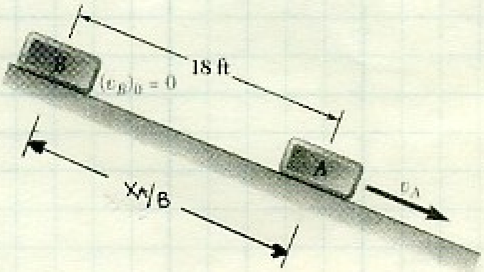
$$t = 3.176 \times 10^{-3} \text{ hr}$$

$$\underline{\underline{t = 11.43 \text{ sec}}}$$

Set #2 – Uniformly Accelerated Motion

2. Boxes are placed on a chute at uniform intervals of time t_R and slide down the chute with uniform acceleration. Knowing that as any box B is released, the preceding box A has already slid 18 ft and that 1 s later they are 30 ft apart, determine

a) the value of t_R,
b) the acceleration of the boxes.



Given: $a_A = a_B = \text{constant}$
boxes are placed every t_R seconds
at $t = t_R$, $x_B = 0$ & $x_A = 18\text{ft}$
at $t = t_R + 1$, $x_A - x_B = 30\text{ ft}$

Find: (a) t_R
(b) a

Solution: $a_{A/B} = a_A - a_B = 0$

$$V_{A/B} = \text{constant} = \frac{\Delta x}{\Delta t} = \left(\frac{30-18}{1}\right)\text{ ft/s} = 12\text{ ft/s}$$

at $t = t_R$,
$$V_{A/B} = V_A - V_B$$
$$12 = V_A - 0$$
$$V_A = 12\text{ ft/s}$$

For block A, use $v_f = v_0 + at$ at $t = t_R$

$$12 = 0 + a t_R$$
$$a = \frac{12}{t_R} \quad \text{—①}$$

For block A, use $x_f = x_0 + v_0 t + \frac{1}{2} a t^2$ at $t = t_R$

$$18 = 0 + 0 t_R + \frac{1}{2} a t_R^2$$
$$a = \frac{36}{t_R^2} \quad \text{—②}$$

Setting ① = ②

$$\frac{12}{t_R} = \frac{36}{t_R^2}$$

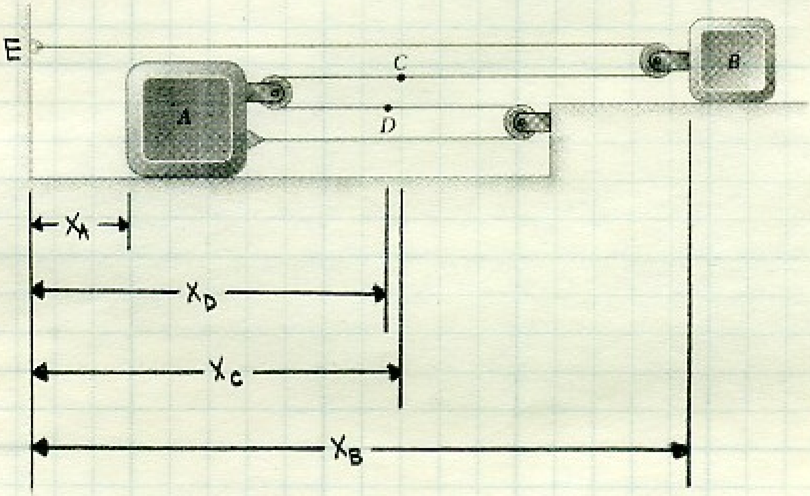
$$\underline{\underline{t_R = 3\text{ sec}}}$$

$$a = \frac{12}{t_R} = \frac{12}{3}$$

$$\underline{\underline{a = 4\text{ ft/s}^2}}$$

Set #2 – Uniformly Accelerated Motion

3. Slider block B moves to the right with a constant velocity of 300 mm/s. Determine
a) the velocity of slider block A,
b) the velocity of portion C of the cable,
c) the velocity of portion D of the cable,
d) the relative velocity of portion C of the cable with respect to slider block A.



Given: $v_B = 300 \text{ mm/s} = \text{constant}$

Find: (a) v_A
(b) v_C
(c) v_D
(d) $v_{C/A}$

Solution:
(a.) Using "conservation of string" idea for total length:

$$x_B + (x_B - x_A) + (-x_A) = \text{constant}$$
$$2x_B - 3x_A = C$$
$$2v_B - 3v_A = 0$$
$$v_A = \frac{2}{3} v_B$$
$$= \frac{2}{3}(300 \text{ mm/s})$$
$$\underline{\underline{v_A = 200 \text{ mm/s}}}$$

(b.) Using "conservation of string" idea for length EC:

$$x_B + x_B - x_C = C$$
$$2x_B - x_C = C$$
$$2v_B - v_C = 0$$
$$v_C = 2v_B$$
$$v_C = 2(300 \text{ mm/s})$$
$$\underline{\underline{= 600 \text{ mm/s}}}$$

Set #2

3. continued.

(c.) for length AD:

$$-x_A + (-x_D) = C$$

$$-v_A - v_D = 0$$

$$\underline{\underline{v_D = -200 \text{ mm/s}}}$$

(d.)

$$v_{C/A} = v_C - v_A$$

$$= 600 - 200$$

$$\underline{\underline{v_{C/A} = 400 \text{ mm/s}}}$$

Set #2 – Uniformly Accelerated Motion

4. Collars A and B start from rest, and collar A moves upward with an acceleration of $3t^2$ inches/s^2. Knowing that collar B moves downward with a constant acceleration and that its velocity is 8 inches/s after moving 32 inches, determine

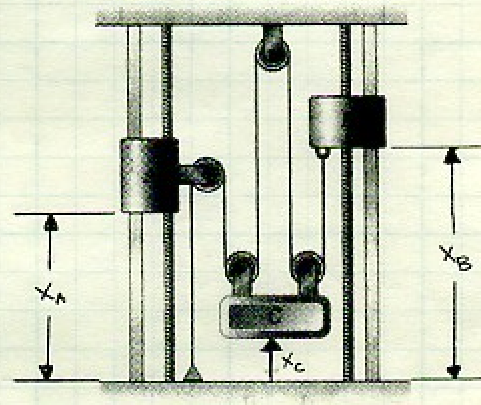
a) the acceleration of block C,
b) the distance through which block C will have moved after 3 s

Given: $v_{A_0} = v_{B_0} = 0$

$a_A = 3t^2$ in/s^2

$a_B = K$

$v_B\big|_{\Delta x_B = -32} = -8$ in/s



Find: (a) a_c

(b.) distance C moves after 3 sec.

Solution: "Conservation of String"

$$x_A + (x_A - x_c) + (-x_c) + (-x_c) + (x_B - x_c) = \text{constant}$$

$$2x_A + x_B - 4x_c = \text{constant}$$

$$2v_A + v_B - 4v_c = 0$$

$$2a_A + a_B - 4a_c = 0$$

$$a_c = 0.5\,a_A + 0.25\,a_B \quad -①$$

(a.) For block B, use $v_f^2 = v_0^2 + 2a(x_f - x_0)$

$$(-8)^2 = 0^2 + 2a_B(-32)$$

$$a_B = \frac{64}{-64} = -1 \text{ in/s}^2$$

a_B into ①

$$a_c = 0.5\,a_A + 0.25\,a_B$$

$$= 0.5(3t^2) + 0.25(-1)$$

$$= (1.5t^2 - 0.25) \text{ in/s}^2$$

Set #2

4. continued.

(b.) To find distance traveled, we need to know if C turns around:

$$a_c = \frac{dv_c}{dt}$$

$$dv_c = a_c\, dt$$

$$\int_0^{v_c} dv_c = \int_0^t (1.5t^2 - 0.25)\, dt$$

$$v_c = \frac{1.5t^3}{3} - 0.25t$$

$$0 = 0.5t^3 - 0.25t$$

$$0 = t(0.5t^2 - 0.25)$$

$$0 = (0.5t^2 - 0.25$$

$$t^2 = \frac{0.25}{0.5}$$

$$t = 0.707 \text{ sec.}$$

$$v_c = \frac{dx_c}{dt}$$

$$\int_0^t (0.5t^3 - 0.25t)\, dt = \int_0^{x_c} x_c$$

$$\frac{0.5t^4}{4} - \frac{0.25t^2}{2} = x_c$$

$$x_c(t) = 0.125t^4 - 0.125t^2$$

$$x_c(0) = 0$$

$$x_c(0.707) = -0.03125$$

$$x_c(3) = 9$$

Distance traveled in 3 sec. =

$$= |x_c(3) - x_c(.707)| + |x_c(.707) + x_c(0)|$$

$$= |9 - (-0.03125)| + |(-0.03125) - 0|$$

$$= \underline{\underline{9.0625 \text{ in}}}$$