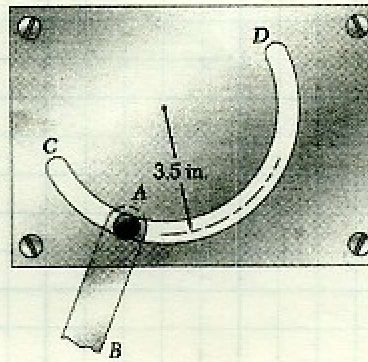
Set #4 – Tangential/Normal & Radial/Transverse Components

1. Pin A, which is attached to link AB, is constrained to move in the circular slot CD. Knowing that at t = 0 the pin starts from rest and moves so that its speed increases at a constant rate of 0.8 inches/s^2, deterimine the magnitude of its total acceleratin when

a) t = 0
b) t = 2 s.



Given: pin A moves in a circular path CD

$\quad$ at $t=0$, $\vec{v}_A = 0$

$\quad\quad \dot{\vec{v}}_A = a = .8 \ ^{in}/_{s^2} = 20 \ ^{mm}/_{s^2}$

$\quad$ radius $= 3.5$ in ≈ 90 mm

Find: (a.) $|\vec{a}|$ at $t=0$

$\quad$ (b.) $|\vec{a}|$ at $t = 2$ sec

Solution: The path is known, so use: $\vec{v} = v\hat{e}_t$, $\vec{a} = \dot{v}\hat{e}_t + \dfrac{v^2}{\rho}\hat{e}_n$

$$\dot{v} = \frac{dv}{dt}$$

$$\int_0^v dv = \int_0^t \dot{v}\,dt$$

$$v(t) = \dot{v}t = 20 \ ^{mm}/_{s^2}\ t$$

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = (20 \ ^{mm}/_{s^2})\,\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

(a.) at $t=0$, $v = 0$

$$\vec{a} = (20 \ ^{mm}/_{s^2})\,\hat{e}_t$$

$$\underline{\underline{|\vec{a}| = 20 \ ^{mm}/_{s^2}}}$$

(b.) at $t=2$

$$v(2) = (20 \ ^{mm}/_{s^2})(2\,\text{sec}) = 40 \ ^{mm}/_{s}$$

$$\vec{a} = (20 \ ^{mm}/_{s^2})\,\hat{e}_t + \frac{(40 \ ^{mm}/_{s})^2}{(90\,\text{mm})}\hat{e}_n$$

$$= (20 \ ^{mm}/_{s^2})\,\hat{e}_t + (17.78 \ ^{mm}/_{s^2})\,\hat{e}_n$$

$$|\vec{a}| = \sqrt{20^2 + 17.78^2}$$

$$= \underline{\underline{26.76 \ ^{mm}/_{s^2}}}$$

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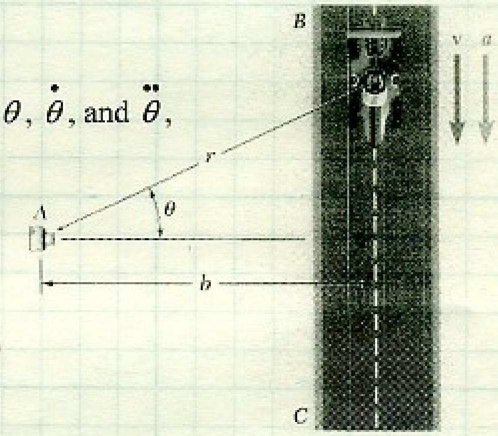
2. To study the performance of a race car, a high speed motion-picture camera is positioned at point A. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightaway BC. Determine

a) the speed of the car in terms of b, θ, and $\dot{\theta}$,

b) the magnitude of the acceleration of the car in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$,

c) the average speed of the car during a 0.5-s interval if, over this interval, the car travels from the position $\theta = 60^\circ$ to the position $\theta = 35^\circ$. (b = 25 m)



Given: Figure

Find: (a) $v(b, \theta, \dot{\theta})$

(b.) $a(b, \theta, \dot{\theta}, \ddot{\theta})$

(c.) V_{AVE} for $\Delta t = (t_f - t_o) = 0.5\ s$

$b = 25\ m$

$\theta_o = 60^\circ$

$\theta_f = 35^\circ$

Solution:

(a)

$$\frac{b}{r} = \cos\theta$$

$$r = \frac{b}{\cos\theta}$$

$$\dot{r} = \frac{\dot{\theta}\, b \sin\theta}{\cos^2\theta} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

$$\ddot{r} = \frac{\cos^2\theta(\dot{\theta}^2 b\cos\theta + b\ddot{\theta}\sin\theta) + 2b\dot{\theta}^2\sin^2\theta\cos\theta}{\cos^4\theta}$$

$$= \frac{b\dot{\theta}^2\cos^3\theta + b\ddot{\theta}\cos^2\theta\sin\theta + 2b\dot{\theta}^2\sin^2\theta\cos\theta}{\cos^4\theta}$$

$$= \frac{b\dot{\theta}^2}{\cos\theta} + \frac{b\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{2b\dot{\theta}^2\sin^2\theta}{\cos^3\theta}$$

Set #4

2. continued

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$= \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}\hat{e}_r + \frac{b\dot{\theta}}{\cos\theta}\hat{e}_\theta$$

$$\vec{v} = \sqrt{v_r^2 + v_\theta^2}$$

$$\vec{v} = \sqrt{\frac{b^2\dot{\theta}^2\sin^2\theta}{\cos^4\theta} + \frac{b^2\dot{\theta}^2}{\cos^2\theta}}$$

$$= \sqrt{\frac{b^2\dot{\theta}^2}{\cos^4\theta}(\overset{1}{\cancel{\sin^2\theta + \cos^2\theta}})}$$

$$\vec{v} = \frac{b\dot{\theta}}{\cos^2\theta}$$

(b.) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$$a_r = \frac{b\dot{\theta}^2}{\cos\theta} + \frac{b\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{2b\dot{\theta}^2\sin^2\theta}{\cos^3\theta} - \frac{b\dot{\theta}^2}{\cos\theta}$$
$$= \frac{b\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{2b\dot{\theta}^2\sin^2\theta}{\cos^2\theta}$$
$$= \frac{b}{\cos\theta}\left(\ddot{\theta}\tan\theta + 2\dot{\theta}^2\tan^2\theta\right)$$

$$a_\theta = \frac{b\ddot{\theta}}{\cos\theta} + \frac{2b\dot{\theta}^2\sin\theta}{\cos^2\theta}$$
$$= \frac{b}{\cos\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$$

$$\vec{a} = \sqrt{a_r + a_\theta}$$

Set #4

2. continued

$$\vec{a} = \sqrt{a_r + a_\theta}$$

$$\vec{a} = \sqrt{\left[\left(\frac{b}{\cos\theta}\right)(\ddot{\theta}\tan\theta + 2\dot{\theta}^2\tan^2\theta)\right]^2 + \left[\left(\frac{b}{\cos\theta}\right)(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)\right]^2}$$

$$= \sqrt{\left(\frac{b^2}{\cos^2\theta}\right)\left\{\left[\tan\theta\,(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)\right]^2 + \left[(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)\right]^2\right\}}$$

$$= \frac{b}{\cos\theta}\sqrt{(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)^2(\tan^2\theta + 1)}$$

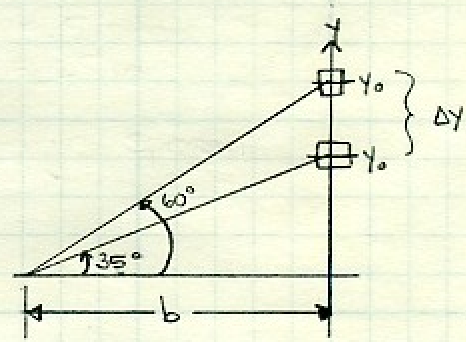
$$= \frac{b(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)}{\cos\theta}\sqrt{\sec^2\theta}$$

$$= \frac{b(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)}{\cos\theta}(\sec\theta)$$

$$\vec{a} = \frac{b(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)}{\cos^2\theta}$$

(c.) Given: $\Delta t = .5$ sec
$b = 25$ m

Find: average speed.



Solution:

$$V_{AVE} = \frac{\Delta y}{\Delta t} = \frac{y_f - y_o}{\Delta t}$$

$$y_o = b\tan 60° = 25\text{ m}\tan 60° = 43.30\text{ m}$$

$$y_f = b\tan 35° = 25\text{ m}\tan 35° = 17.51\text{ m}$$

$$V_{AVE} = \frac{\Delta y}{\Delta t} = \frac{y_f - y_o}{\Delta t} = \left(\frac{43.30 - 17.51}{.5}\right)\frac{m}{s}$$

$$V_{AVE} = 51.6\ \text{m/s} \downarrow = 185.7\ \text{km/hr} \downarrow$$

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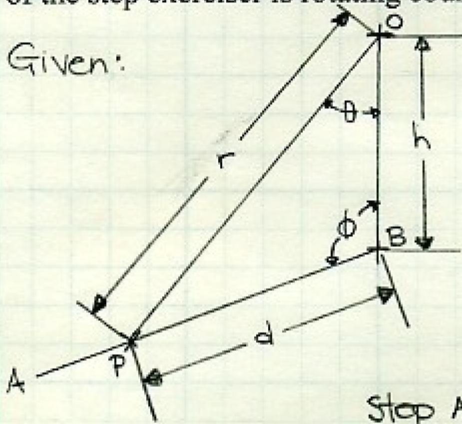
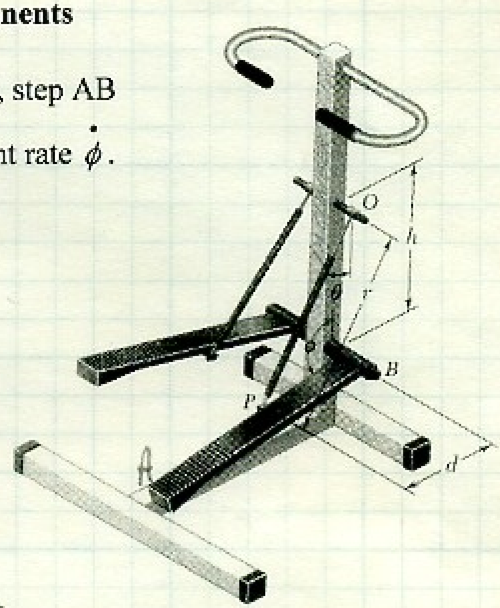
3. Show that $\dot{r} = h\dot{\phi}\sin\theta$ knowing that at the instant shown, step AB of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

Given:





Step AB is rotating
CCW at constant rate
$\dot{\phi}$ at instant shown.

Find: $\dot{r}$

Solution:

Law of Cosines: $r^2 = h^2 + d^2 + (-2hd\cos\phi)$

h^2, d^2: constants; $(-2hd\cos\phi)$: variable

$$\frac{d}{dt}: \quad 2r\dot{r} = 0 + 0 + 2hd\sin\phi\,\dot{\phi}$$

$$\dot{r} = \frac{2hd\dot{\phi}\sin\phi}{2r} \quad \text{—①}$$

Note: We need to get rid of r. Find r in terms of other variables. How? Law of Sines!

Law of Sines: $\dfrac{r}{\sin\phi} = \dfrac{d}{\sin\theta}$

$$r = d\,\frac{\sin\phi}{\sin\theta} \quad \text{—②}$$

Substitute ② into ①.

$$\dot{r} = \frac{hd\dot{\phi}\sin\phi\sin\theta}{d\sin\phi}$$

$$\underline{\underline{\dot{r} = h\dot{\phi}\sin\theta}}$$