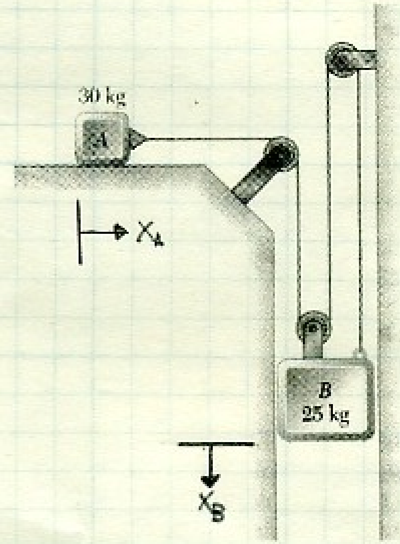
Set #5 – Equations of Motion

1. The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficients of friction between block A and the horizontal surface are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine:

a) The acceleration of each block.

b) The tension in the cable.



Given: Figure

For block A, $\mu_s = 0.25$

$\mu_k = 0.20$

$v_{A_0} = 0$

$v_{B_0} = 0$

massless, frictionless pulley.

Find: (a.) $\vec{a}_A$

$\vec{a}_B$

(b.) T

Solution:

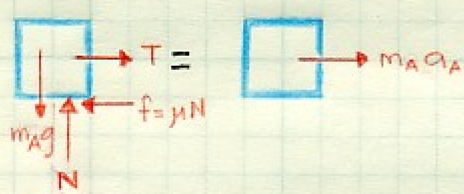
Note: $-x_A + x_B + x_B + x_B = C$

$-x_A + 3x_B = 0$

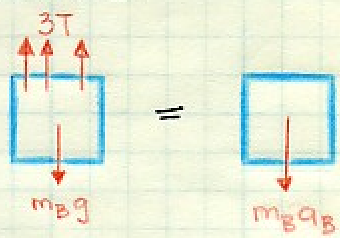
$-v_A + 3v_B = 0$

$a_A = 3a_B \quad$ — (1)

Block A



Block B



Assume Motion:

Block A → $\Sigma F_y = m_A a_A$ $\qquad$ Note $a_{Ay} = 0$, $\therefore \Sigma F_y = 0$

$= 0$

$m_A g - N = 0$

$N = m_A g$

$= 294.3\text{N}$

$\Sigma F_x = m_A a_A$

$T - \mu_k N = m_A a_A$

$T - (0.2)(294.3\text{ N}) = (30\text{ kg})a_A$

$T - 58.86 = (30\text{ kg})a_A \quad$ — (2)

Block B → $\Sigma F_y = m_B a_B$

$m_B g - 3T = m_B a_B$

$245.25 - 3T = m_B a_B \quad$ — (3)

Set #5

1. continued

Sub 1 into 2 : $T - 58.86 = (30\ kg)\ 3a_B$

3 : $-3T + 245.25 = (25\ kg)\ a_B$

$$3T - 3(58.86) = 3(30\ kg)\ 3a_B$$
$$-3T + 245.25 = (25\ kg)\ a_B$$

$$3T - 176.58 = (270\ kg)\ a_B$$
$$-3T + 245.25 = (25\ kg)\ a_B$$
$$0 + 68.67 = (295\ kg)\ a_B$$

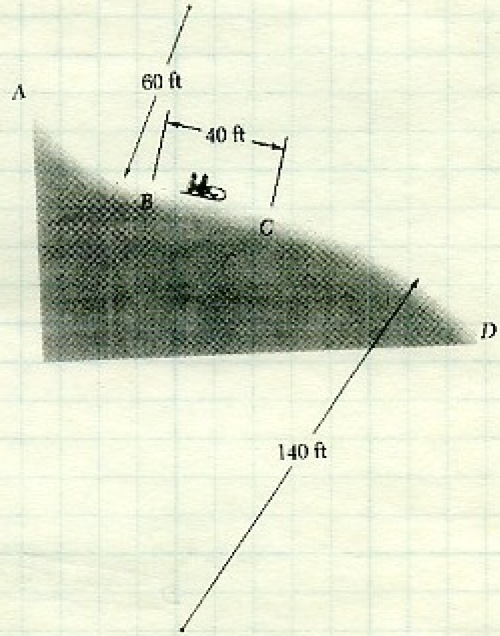
$$a_B = \frac{68.67\ kg \cdot m}{295\ kg \cdot s^2} = \underline{\underline{0.233\ m/s^2 \downarrow}}$$

$$a_A = 3a_B = 3(0.233) = \underline{\underline{0.699\ m/s^2 \rightarrow}}$$

$$-3T + m_B g = m_B a_B$$
$$T = \frac{(25)a_B - 245.25}{-3} = \frac{(25)(0.233) - 245.25}{-3} = \underline{\underline{79.8\ N}}$$

Set #5 – Equations of Motion

2. The portion of a toboggan run shown is contained in a vertical plane. Sections AB and CD have radii of curvature as indicated, and section BC is straight and forms an angle of 20° with the horizontal. Knowing that the coefficient of kinetic friction between a sled and the run is 0.10 and that the speed of the sled is 25 ft/s at B, determine the tangential component of the acceleration of the sled
a) just before it reaches B, and
b) just after is passes C.



Given: Figure
BC is 20° from horizontal

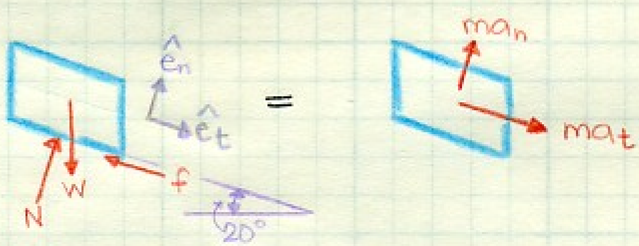
B, 20°, C

$\mu_K = .10$
at B, $v = 25$ ft/s

Find: (a) a_t at B^- (just before B)
(b.) a_t at C^+ (just after C)

Solution: $\vec{v} = v\hat{e}_t$
$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$

(a.)



• $\Sigma F_n = ma_n$

$N - W\cos 20° = m\frac{v^2}{\rho}$

$N = W\cos 20° + m\frac{v^2}{\rho}$

$N = mg\cos 20° + \frac{mv^2}{\rho}$ —①

Set #5

2. continued

- $\sum F_t = ma_t$

$$mg \sin 20° - f = ma_t$$
$$mg \sin 20° - \mu_k N = ma_t \quad \text{— ②}$$

① into ②,

$$mg \sin 20° - \mu_k \left(mg \cos 20° + \frac{mv^2}{\rho} \right) = ma_t$$

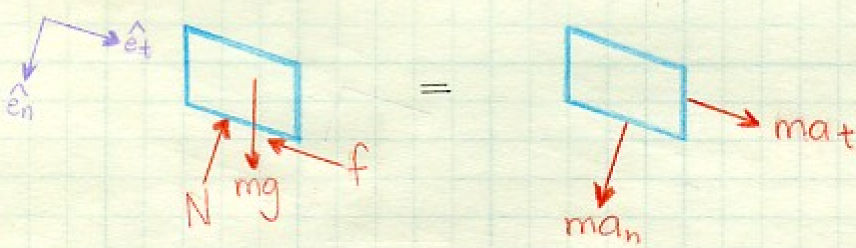
$$\cancel{m}g \sin 20° - \mu_k \cancel{m}g \cos 20° - \mu_k \frac{\cancel{m}v^2}{\rho} = \cancel{m}a_t$$

$$\rightarrow a_t = g \sin 20° - \mu_k g \cos 20° - \mu_k \frac{v^2}{\rho}$$

$$a_t = (32.2) \sin 20° - (.1)(32.2) \cos 20° - (.1) \frac{(25)^2}{60}$$

$$\underline{\underline{a_t = 6.95 \ \text{ft/s}^2}} \ \text{at } B^-$$

(b.) At C^+



- $\sum F_n = ma_n$

$$mg \cos 20° - N = \frac{mv^2}{\rho}$$

$$N = mg \cos 20° - \frac{mv^2}{\rho} \quad \text{— ③}$$

Set #5

2. continued

$$\Sigma F_t = ma_t$$

$$mg \sin 20° - f = ma_t$$
$$mg \sin 20° - \mu_K N = ma_t \quad — ④$$

③ into ④,

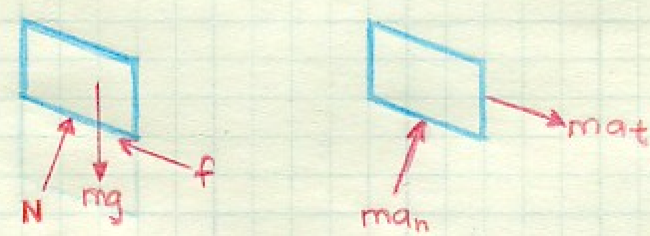
$$mg \sin 20° - \mu_K \left(mg \cos 20° - \frac{mv^2}{\rho} \right) = ma_t$$
$$\cancel{m}g \sin 20° - \mu_K \cancel{m}g \cos 20° + \mu_K \frac{\cancel{m}v^2}{\rho} = \cancel{m}a_t$$

$$a_t = g \sin 20° - \mu_K g \cos 20° + \mu_K \frac{v^2}{\rho} \quad — ⑤$$

Note: We do not know the value of v at C^+ in ⑤.

To find v: $\underline{v_{c^-}^2 = v_{B^+}^2 + 2a_t d}$ — ⑥

Now, we must find a_t at B^+. To do so, we have to create a FBD of the object.



$$\Sigma F_n = ma_n$$
$$N - mg \cos \theta = ma_y = 0$$
$$N = mg \cos \theta$$

$$\Sigma F_t = ma_t$$
$$mg \sin \theta - f = ma_t$$
$$\cancel{m}g \sin \theta - \mu_K (\cancel{m}g \cos \theta = \cancel{m}a_t$$

$$a_t = g \sin \theta - \mu_K g \cos \theta$$
$$= 7.987 \text{ ft/s}^2$$

Set #5

2. continued

Knowing the value of a_t, we can solve for v_{c^-} using equation 6.

$$v_{C^-}^2 = v_{B^+}^2 + 2a_t d$$

$$v_{C^-} = \sqrt{(25)^2 + (2)(7.98)(40)}$$
$$= 35.56\ m/s$$

We no longer have unknowns in 5. We can now solve for a_t at C^+

$$a_t = g \sin 20° - \mu_K g \cos 20° + \mu_K \frac{v^2}{\rho}$$
$$= (32.2) \sin 20° - (0.1)(32.2) \cos 20° + (0.1) \frac{(35.56)^2}{140}$$
$$\underline{\underline{= 8.89\ ft/s^2 \text{ at } C^+}}$$

Set #5 – Equations of Motion

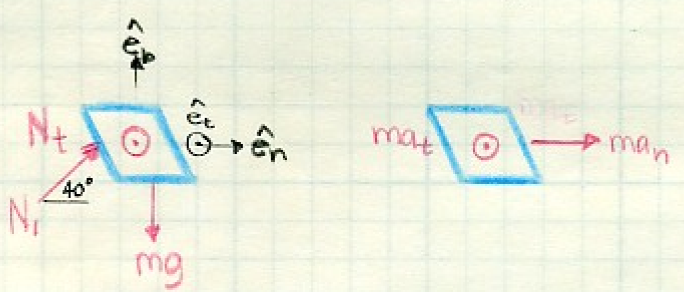
3. A small, 300-g collar D can slide on portion AB of a rod which is bent as shown. Knowing that $\alpha = 40^{\circ}$ and that the rod rotates about the vertical AC at a constant rate of 5 rad/s, determine the value of r for which the collar will not slide on the rod if the effect of friction between the rod and the collar is neglected.

Given: $m_D = 300\,g$
$\omega = 5$ rad/s
$\mu = 0$ (frictionless)

Find: r for no slip

Solution: If there is no slip, D is moving in a circular path of radius r about the axis AC.

→ Use $\hat{e}_t$ and $\hat{e}_n$



$$\Sigma F_b = ma_b \rightarrow N_1 \sin 40^{\circ} - mg = 0$$

$$N_1 = \frac{mg}{\sin 40^{\circ}} \quad -①$$

$$\Sigma F_n = ma_n \rightarrow N_1 \cos 40^{\circ} = m\frac{v^2}{r}$$

$$N_1 = \frac{m r \omega^2}{\cos 40^{\circ}} \quad -②$$

Set ② = ① → $\frac{\cancel{m}g}{\sin 40^{\circ}} = \frac{\cancel{m}r\omega^2}{\cos 40^{\circ}}$

$$\frac{g}{\sin 40^{\circ}} = \frac{r\omega^2}{\cos 40^{\circ}}$$

$$r = \frac{g \cos 40^{\circ}}{\omega^2 \sin 40^{\circ}}$$

$$= \frac{g}{\omega^2 \tan 40^{\circ}}$$

$$= \frac{9.81}{(5)^2 \tan 40^{\circ}}$$

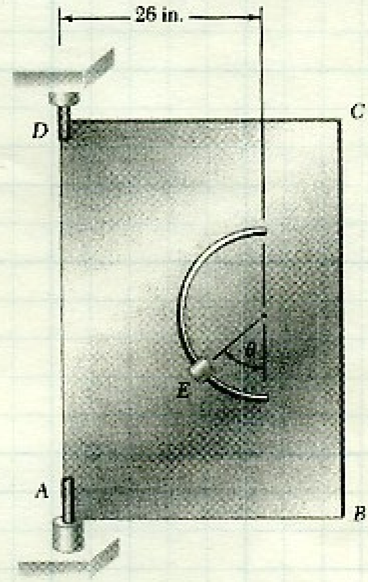
$$= \underline{\underline{0.468\ m}}$$

Set #5 – Equations of Motion

4. A semicircular slot of 10-inch radius is cut in a flat plate which rotates about the vertical AD at a constant rate of 14 rad/s. A small, 0.8-lb block E is designed to slide in the slot as the plate rotates. Knowing that the coefficients of friction are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine whether the block will slide in the slot if it is released in the position corresponding to

a) $\theta = 80^\circ$
b) $\theta = 40^\circ$

Also determine the magnitude and the direction of the friction force exerted on the block immediately after it is released.

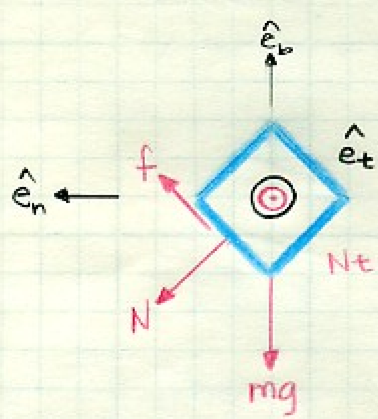


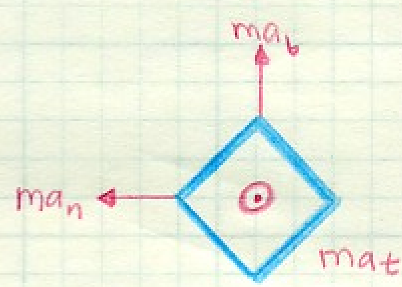
Given: $\omega = 14 \text{ rad/s}$ (constant)
$m_E g = 0.8 \text{ lb}_f$
$\mu_s = .35$
$\mu_k = .25$

Find: (a) Will block slide if released at $\theta = 80^\circ$
(b) will block slide if released at $\theta = 40^\circ$
(c) $\vec{f}$ immediately after release.

Solution: Assuming NO slip, the path is a circle of

$$r = (26 - 10 \sin\theta)\left(\frac{1}{12}\right) \text{ ft} \qquad \text{around AD}$$





$$a_n = \frac{mv^2}{r}$$

$$a_n = \frac{mr^2v^2}{r}$$

$$a_n = mrv^2 \quad \text{—(1)}$$

Set # 5

4. continued

$$\Sigma F_n = ma_n \rightarrow f\cos\theta + N\sin\theta = ma_n$$

plug in ①

$$f\cos\theta + N\sin\theta = mr\omega^2 \quad ②$$

$$\Sigma F_b = ma_b \rightarrow f\sin\theta - N\cos\theta = mg \quad ③$$

$$\begin{aligned} f\cos\theta + N\sin\theta &= mr\omega^2 \\ f\sin\theta - N\cos\theta &= mg \end{aligned}$$

$\rightarrow$ ② $\cos\theta$ + ③ $\sin\theta$:

$$\begin{aligned} f\cos^2\theta + N\sin\theta\cos\theta &= mr\omega^2\cos\theta \\ f\sin^2\theta - N\sin\theta\cos\theta &= mg\sin\theta \\ \hline f(1) &= mr\omega^2\cos\theta + mg\sin\theta \end{aligned}$$

$$f = mr\omega^2\cos\theta + mg\sin\theta \quad ④$$

- We are trying to find if the block will slide. By using ③ and ④, we are able to find $\frac{f}{N}$.

  If $\frac{f}{N} \le \mu_s$, the block will not slide.

- To find $\frac{f}{N}$, we will have to solve ③ for N and subsequently substitute f (equ. ④) in that equation. We can then use that new equation to find $\frac{f}{N}$.

$\longrightarrow$ Rewriting ③, $N = \dfrac{f\sin\theta - mg}{\cos\theta}$

Plug in ④,

$$N = \frac{(mr\omega^2\cos\theta + mg\sin\theta)\sin\theta - mg}{\cos\theta}$$

$$= \frac{mr\omega^2\sin\theta\cos\theta + mg\sin^2\theta - mg}{\cos\theta}$$

We can now use this N and ④ to find $\frac{f}{N}$.

Set #5

4. continued

$$\frac{f}{N} = \frac{mr\omega^2\cos\theta + mg\sin\theta}{\left(\dfrac{mr\omega^2\sin\theta\cos\theta + mg\sin^2\theta - mg}{\cos\theta}\right)}$$

$$= \frac{(\cancel{m}r\omega^2\cos\theta + \cancel{m}g\sin\theta)\cos\theta}{\cancel{m}r\omega^2\sin\theta\cos\theta + \cancel{m}g\sin^2\theta - \cancel{m}g}$$

$$= \frac{r\omega^2\cos^2\theta + g\sin\theta\cos\theta}{r\omega^2\sin\theta\cos\theta + g\sin^2\theta - g}$$

$\rightarrow r = 26 - 10\sin\theta$

$$\frac{f}{N} = \frac{(26-10\sin\theta)\,\omega^2\cos^2\theta + g\sin\theta\cos\theta}{(26-10\sin\theta)\,\omega^2\sin\theta\cos\theta + g\sin^2\theta - g}$$

- For $\omega = 14$, $\theta = 40°$, $\frac{f}{N} = 1.2123$.

  This is not $\leq .35$; the block will slip.

- For $\omega = 14$, $\theta = 80°$, $\frac{f}{N} = .18683$.

  This is $\leq .35$; the block will not slip.