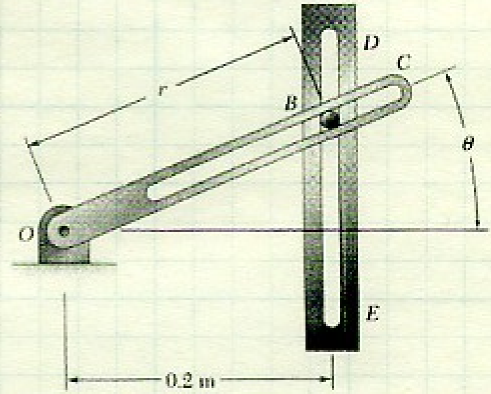
Set #6 – Angular Momentum

1. The 100-g pin B slides along the slot in the rotating arm OC and along the slot DE which is cut in a fixed horizontal plate. Neglecting friction and knowing that rod OC rotates at the constant rate $\dot{\theta}_o$=12 rad/s, determine for any given value of θ

a) the radial and transverse components of the resultant force **F** exerted on pin B,

b) the forces **P** and **Q** exerted on pin B by rod OC and the wall of slot DC, respectively.



Given: $m_B = 0.100$ kg

$\omega_{oc} = \dot{\theta}_o = 12$ rad/s = constant

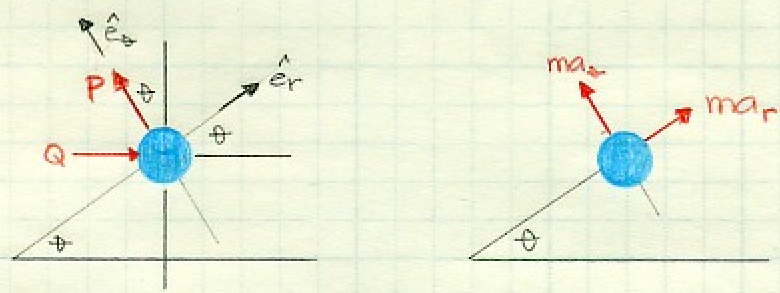
frictionless

Find: (a.) R_r, R_θ - the components of $\vec{R}$ on B from slots.

(b.) $\vec{P}$: force of OC on B

$\vec{Q}$: force of DE on B

Solution: FBD = IND



$$\Sigma F_r = ma_r$$
$$= m(\ddot{r} - r\dot{\theta}^2) \quad (1)$$

$$\Sigma F_r = R_r = Q\cos\theta$$

$$Q\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = ma_\theta$$
$$= m(r\cancel{\ddot{\theta}}^{\,0} + 2\dot{r}\dot{\theta}) \quad (2)$$

$$\Sigma F_\theta = R_\theta = P - Q\sin\theta$$

$$P - Q\sin\theta = 2m\dot{r}\dot{\theta}$$

Set #6

1. continued

(a.) We know $\dot{\theta} = 12 \Rightarrow \ddot{\theta} = 12t$

So we see we need $r, \dot{r}, \ddot{r}$ as $f(\theta)$

$$r = \frac{0.2}{\cos\theta} \quad \text{(from geometry)} \qquad — ③$$

$$\dot{r} = \frac{dr}{dt} = \frac{0.2 \sin\theta \, \dot{\theta}}{\cos^2\theta}$$
$$= \frac{2.4 \sin\theta}{\cos^2\theta} \qquad — ④$$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{2.4(\cos^3\theta \, \dot{\theta} + \sin^2\theta \, 2\cos\theta \, \dot{\theta})}{\cos^4\theta}$$
$$= \frac{2.4\dot{\theta}(\cos^2\theta + 2\sin^2\theta)}{\cos^3\theta}$$
$$= \frac{28.8}{\cos\theta}(1 + 2\tan^2\theta) \qquad — ⑤$$

Substitute ③ and ⑤ into ①

$$\Rightarrow R_r = Q\cos\theta = m\left[\frac{28.8}{\cos\theta}(1 + 2\tan^2\theta) - \frac{0.2}{\cos\theta}(12)^2\right]$$

$$R_r = m\frac{28.8}{\cos\theta}\left[1 + 2\tan^2\theta - 1\right]$$
$$= \frac{28.8\,m}{\cos\theta}(2\tan^2\theta)$$
$$= 57.6\,m\,\frac{\sin^2\theta}{\cos^3\theta}$$

Substitute ③ and ④ into ②

$$\Rightarrow R_\theta = P - Q\sin\theta = 2m\left(\frac{2.4\sin\theta}{\cos^2\theta}\right)$$
$$= 57.6\,m\,\frac{\sin\theta}{\cos^2\theta}$$

Set #6

1. continued

(b) $R_r = Q\cos\theta$

$$\Rightarrow Q = \frac{R_r}{\cos\theta}$$

$$= \frac{57.6\ m\ \sin^2\theta}{\cos^4\theta}$$

$$R_\theta = P - Q\sin\theta$$

$$\Rightarrow P = R_\theta + Q\sin\theta$$

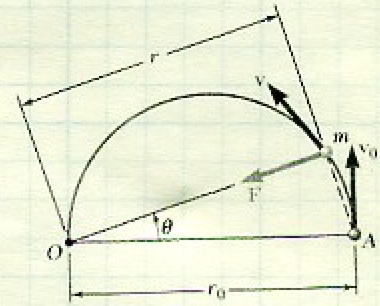
$$= \frac{57.6\ m\ \sin\theta}{\cos^2\theta} + \frac{57.6\ m\ \sin^3\theta}{\cos^4\theta}$$

$$= \frac{57.6\ m\ \sin\theta}{\cos^4\theta}\left(\cos^2\theta + \sin^2\theta\right)$$

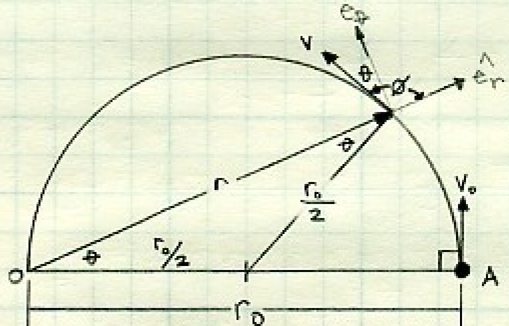
$$= \frac{57.6\ m\ \sin\theta}{\cos^4\theta}$$

Set #6 – Angular Momentum

2. A particle of mass m is projected from point A with an initial velocity $\mathbf{v}_0$ perpendicular to the line OA and moves under a central force **F** along a semicircular path of diameter OA. Observing that $r = r_0 \cos\theta$, show that the speed of the particle is $v = v_0/\cos^2\theta$.



Given: A is moving under a central force along semi-circle OA



Find: $v(\theta)$

Solution: Since A is moving under a central force
→ angular momentum is conserved.

⇒ Use Principle of Conservation of Angular Momentum:

$$\vec{h} = \vec{r}_0 \times \vec{V}_0 = \vec{r} \times \vec{v}$$

$$h = r_0 V_0 \cancel{\sin 90^\circ} = rv \sin\phi$$
$$= rv\cos\theta$$
$$r_0 V_0 = rv\cos\theta$$

$$v = \frac{r_0 V_0}{r\cos\theta} \quad \text{—(1)}$$

Note: from geometry, $r = 2\left(\frac{r_0}{2}\cos\theta\right)$

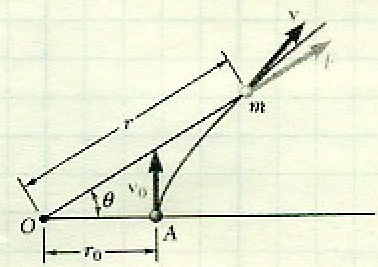
$$r = r_0\cos\theta \quad \text{—(2)}$$

Substitute (2) into (1), $v = \frac{r_0 V_0}{r_0 \cos^2\theta}$

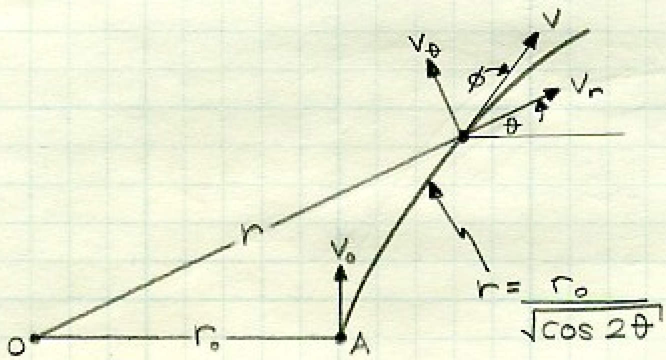
$$\underline{\underline{v = \frac{V_0}{\cos^2\theta}}}$$

Set #6 – Angular Momentum

3. A particle of mass m is projected from point A with an initial velocity $\mathbf{v}_o$ perpendicular to line OA and moves under a central force **F** directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_o / \sqrt{\cos 2\theta}$, express the radial and transverse components of the velocity **v** of the particle as functions of θ.



Given: A is moving under a central force.



Find: v_r
v_θ

Solution: Use h = constant.

Initially, $h = r_o v_o \sin 90°$
$= r_o v_o$ — (1)

At any time, t, $h = r v \sin\phi = r v_\theta$ — (2)

Set (1) = (2) → $r_o v_o = r v_\theta$

$$v_\theta = \frac{r_o v_o}{r} = \frac{\cancel{r_o} v_o}{\cancel{r_o}/\sqrt{\cos 2\theta}}$$

$$\underline{\underline{v_\theta = v_o \sqrt{\cos 2\theta}}}$$

Set #6

3. continued.

$$V_r = \dot{r} = \frac{-r_0\left[\cancel{1/2}(\cos 2\theta)^{-1/2}\right](-\sin 2\theta)\cancel{(2)}\,\dot{\theta}}{\cos 2\theta}$$

$$= \frac{+\dot{\theta}\, r_0\, (\cos 2\theta)^{-1/2}\ \sin 2\theta}{\cos 2\theta}$$

$$= \frac{\theta\, r_0\, \sin 2\theta}{(\cos 2\theta)^{3/2}} \quad \text{— } 3$$

Recall, $h = r_0 V_0 = r(r\dot{\theta})$
$\qquad\qquad\qquad\quad = r^2\dot{\theta}$

$$\dot{\theta} = \frac{r_0 V_0}{r^2} \quad \text{— } 4$$

Substitute 3 into 4

$$V_r = \left(\frac{r_0 V_0}{r^2}\right)\left(\frac{r_0 \sin 2\theta}{(\cos 2\theta)^{3/2}}\right)$$

$$= \left(\frac{\cancel{r_0} V_0}{\cancel{r_0^2}/\cos 2\theta}\right)\left(\frac{\cancel{r_0} \sin 2\theta}{(\cos 2\theta)^{3/2}}\right)$$

$$V_r = \frac{V_0 \sin 2\theta}{(\cos 2\theta)^{1/2}}$$