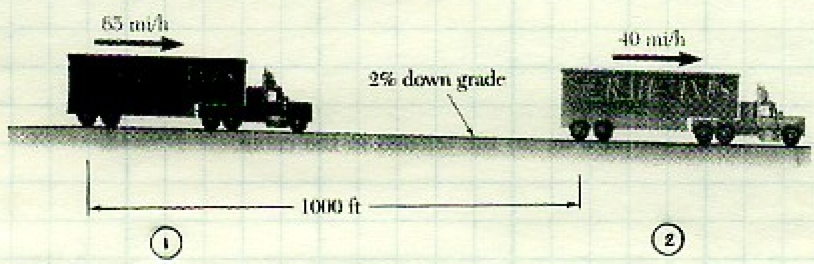
Set #7 – Work, Energy, Power

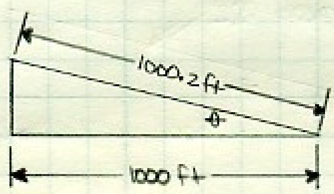
1. A trailer truck enters a 2% downhill grade traveling at 65mph and must slow down to 40mph in 1000ft. The cab weighs 4000 lb and the trailer 12,000 lb. Determine

a) the average braking force that must be applied,

b) the average force exerted on the coupling between a and trailer if 70 % of the braking force is supplied by the trailer of 30 % by the cab.



Given:



$$\tan\theta = 0.02$$

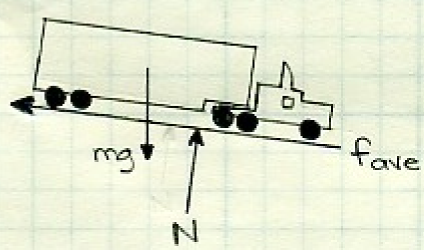
$$v_1 = 65\ \text{mi/h} = 95.33\ \text{ft/sec}$$
$$v_2 = 40\ \text{mi/h} = 58.67\ \text{ft/sec}$$

Find: (a.) f_{ave} for braking

(b.) f in coupling if 70% of f_{ave} comes from trailer?

Solution:

(a) Forces acting during braking (between 1 and 2)



$$T_1 + U_{1\to 2} = T_2$$

$$T_1 = \tfrac{1}{2} m v_1^2 = \frac{1}{2}\left(\frac{16000\ \text{lb}_f}{32.2\ \text{ft/s}^2}\right)\left(95.33\ \text{ft/sec}\right)^2$$

$$T_2 = \tfrac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{16000}{32.2}\right)\left(58.67\right)^2$$

$$U_{1\to 2} = -(f\cos\alpha)\,\Delta x + mg\,\Delta y = -f_{ave}(1000.2) + mg(1000)(0.02)$$

Set #7

1. continued

$$T_1 + U_{1\to 2} = T_2$$

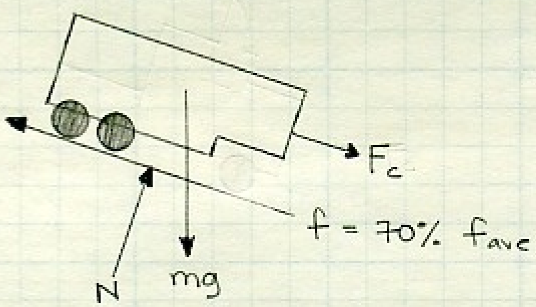
$$\left(\frac{1}{2}\right)\left(\frac{16000\ lbs}{32.2\ ft/s^2}\right)\left(95.33\ ft/s\right)^2 + mg(1000\ ft)(0.02) - f_{ave}\ (1000.2\ ft)$$
$$= \left(\frac{1}{2}\right)\left(\frac{16000\ lbs}{32.2\ ft/s^2}\right)\left(58.62\ ft\right)^2$$

$$f_{ave} = \frac{\frac{1}{2}\left(\frac{16000\ lbs}{32.2\ ft/s^2}\right)\left[(95.33\ ft)^2 - (58.62\ ft)^2\right] + (16000\ lb)(20\ ft)}{1000.2\ ft.}$$

$$= \frac{\left[(1.401 \times 10^6) + 3.2 \times 10^6\right]}{1000.2\ ft}$$

$$= \underline{\underline{1,723\ lbs.}}$$

(b.)



$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2} m_T v_1^2 + \left[m_T g \Delta y - (.70) f_{ave}\ \Delta x + F_c\ \Delta x\right] = \frac{1}{2} m v^2$$

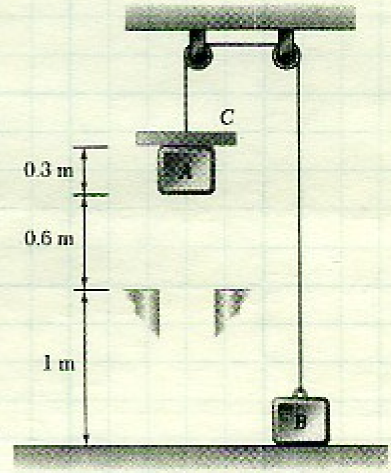
$$\frac{1}{2}\left(\frac{12000}{32.2}\right)(95.33)^2 + \left[(12000)(20) - (.70)(1723)(1000.2)\right.$$
$$\left. + F_c\ (1000.2)\right] = \frac{1}{2}\left(\frac{12000}{32.2}\right)(58.67)^2$$

$$F_c = \frac{\frac{1}{2}\left(\frac{12000}{32.2}\right)(58.67^2 - 95.33^2) - (12000)(20) + (.70)(1723)(1000.2)}{1000.2}$$

$$= \underline{\underline{-85.8\ lbs \rightarrow \text{Compression}}}$$

Set #7 – Work, Energy, Power

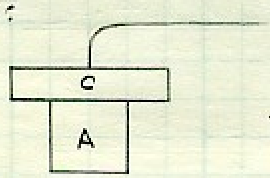
2. Two blocks A and B, of mass 4 kg and 5 kg respectively, are connected by a cord which passes over pulleys as shown. A 3-kg collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9 m, collar C is removed and blocks A and B continue to move. Determine the speed of block A just before it strikes the ground.



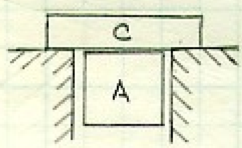
Given: $m_A = 4$ kg $m_C = 3$ kg
$m_B = 5$ kg
released from rest.

Find: v_A just before it hits the ground

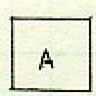
Solution:

1 

V_1 is velocity at position 1

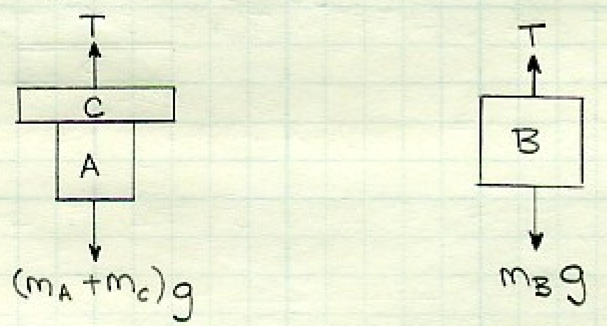
2 

V_2 is velocity at position 2

3 

V_3 is velocity at position 3

Considering motion between positions 1 and 2



$T_{A_1} + U_{1\to2} = T_{A_2}$ $T_{B_1} + U_{1\to2} = T_{B_2}$

Set #7

2. continued

For A: $T_{A_1} = \frac{1}{2}(m_A + m_c)v_1^2 = 0$

$T_{A_2} = \frac{1}{2}(m_A + m_c)v_2^2$

$U_{1\to2} = (m_A + m_c)gx - Tx$

$T_{A_1} + U_{1\to2} = T_{A_2}$

$0 + (m_A + m_c)gx - Tx = \frac{1}{2}(m_A + m_c)v_2^2$ — 1

For B: $T_{B_1} = \frac{1}{2}m_B v_1^2 = 0$

$T_{B_2} = \frac{1}{2}m_B v_2^2$

$U_{1\to2} = -m_B g + Tx$

$T_{B_1} + U_{1\to2} = T_{B_2}$

$0 - m_B gx + Tx = \frac{1}{2}m_B v_2^2$ — 2

Add equations 1 and 2 together.

$(m_A + m_c)gx - \cancel{Tx} - m_B gx + \cancel{Tx} = \frac{1}{2}(m_A + m_c)v_2^2 + \frac{1}{2}m_B v_2^2$

$2(m_A + m_c)gx - 2m_B gx = (m_A + m_c)v_2^2 + m_B v_2^2$

$2gx(m_A + m_c - m_B) = v_2^2(m_A + m_c + m_B)$

$$v_2 = \sqrt{\frac{2gx(m_A + m_c - m_B)}{(m_A + m_c + m_B}}$$

$$= \sqrt{\frac{2(9.81\ \mathrm{m/s^2})(0.9\ \mathrm{m})(4\ \mathrm{kg} + 3\ \mathrm{kg} - 5\ \mathrm{kg})}{(4\ \mathrm{kg} + 3\ \mathrm{kg} + 5\ \mathrm{kg})}}$$

$\underline{\underline{v_2 = 1.7155\ \mathrm{m/s}}}$

Set #7.

2. continued

$$T_2 + U_{2\to 3} = T_3$$

$$\tfrac{1}{2}(m_A+m_B)v_2^2 + m_A g(.7) - m_B g(.7) = \tfrac{1}{2}(m_A+m_B)v_3^2$$

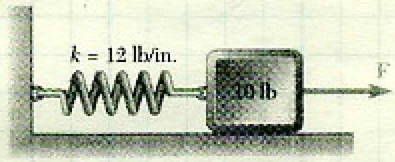
$$v_3 = \sqrt{\frac{2\left[\tfrac{1}{2}(9)(1.716)^2 + (4)(9.81)(.7) - (5)(9.81)(.7)\right]}{9}}$$

$$\underline{\underline{v_3 = 1.190 \text{ m/s}}}$$

Set #7 – Work, Energy, Power

3. A 10-lb block is attached to an unstretched spring of constant k = 12 lb/in. The coefficients of static and kinetic friction between the block and the plane are 0.60 and 0.40 respectively. If a force **F** is slowly applied to the block until the tension in the spring reaches 20 lb and then suddenly removed, determine

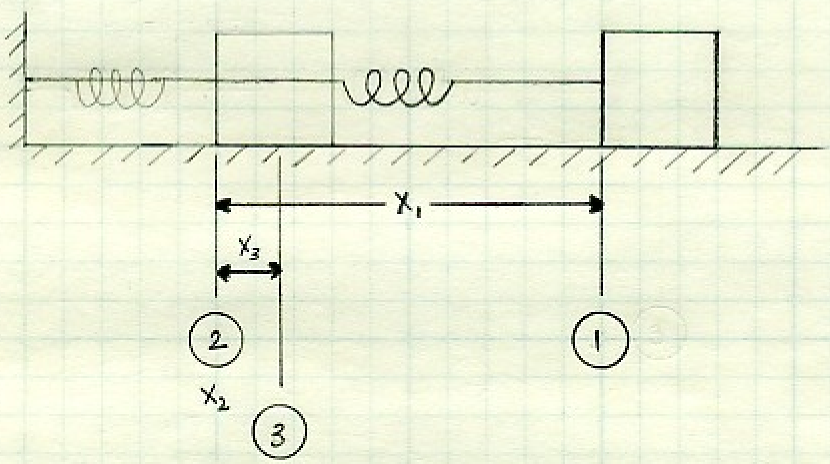
a) the velocity of the block as it returns to its initial position,
b) the maximum velocity achieved by the block.



Given: $k = 12$ lb/in
$\mu_s = 0.60$
$\mu_K = 0.40$
$\vec{F}$ is slowly appied until $T = 20$ lb,
then suddenly removed.

Find: (a.) v at initial position
(b.) v_{max} of block

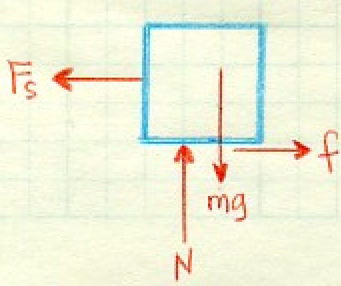
Solution:



(a.)
Position (1) is just before $\vec{F}$ is released and $v=0$

Position (2) is when the block passes through $x=0$ (the unstretched length) as it moves in the $-x$ direction.

Between (1) and (2),

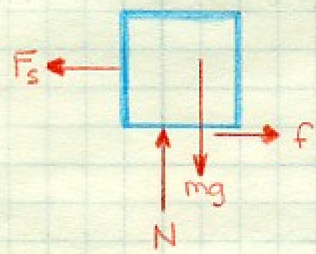


F is removed

F_e and f are the only forces that do work.

Set #7

3. continued



$x_2 = 0$

$x_1 \rightarrow$ at (1), $F_e = 20$ lb

$$F = kx$$
$$20\text{lb} = 12\ x_1$$
$$x_1 = 1.67 \text{ in}$$

$$T_1 + U_{1\rightarrow 2} = T_2$$

$$T_1 = \tfrac{1}{2} m v_1^2 = 0$$
$$T_2 = \tfrac{1}{2} m v_2^2$$

$$\begin{aligned} U_{1\rightarrow 2} &= (\tfrac{1}{2} k x_1^2 - \tfrac{1}{2} k x_2^2) - f x_1 \\ &= \tfrac{1}{2} k (x_1^2 - x_2^2) - \mu_k N x_1 \\ &= \tfrac{1}{2} (12\ \text{lb/in}) [(1.67 \text{ in})^2 - 0^2] - (.4)(10 \text{ lbs})(1.67 \text{ in}) \\ &= 10 \text{ in} \cdot \text{lb} \\ &= .833 \text{ ft} \cdot \text{lb} \end{aligned}$$

$$0 + .833 \text{ ft} \cdot \text{lb} = \tfrac{1}{2} \left(\frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} \right) v_2^2$$

$$\underline{\underline{v_2 = 2.316 \text{ ft/s}}}$$

(b.) Position (3) is where the velocity is maximum.

$a = \frac{dv}{dt}$ $\quad v_{max}$ occurs when $a = 0$

$a = 0$ when forces are balanced in the x-dir.

$$\Sigma F = 0 \rightarrow F_e = f$$
$$k x_3 = \mu_k mg$$
$$x_3 = \frac{\mu_k mg}{k}$$
$$= .33 \text{ in}$$
$$= 0.0278 \text{ ft}$$

Set #7

3. continued

$$T_1 + U_{1\to3} = T_3$$

$$T_1 = \tfrac{1}{2} m v_1^2 = 0$$
$$T_3 = \tfrac{1}{2} m v_{max}^3$$

$$U_{1\to3} = \tfrac{1}{2} k(x_1^2 - x_3^2) - f(x_1 - x_3)$$
$$= \tfrac{1}{2}(144 \text{ lb/ft})(.1389^2 - 0.0278^2) - 4\text{lb}(.1389 - 0.0278)$$
$$= .889 \text{ ft}\cdot\text{lbs}$$

$$v_{max} = \sqrt{2(.889 \text{ ft}\cdot\text{lbs})\frac{(32.2 \text{ ft/s}^2)}{10 \text{ lb}}}$$
$$= \underline{\underline{2.39 \text{ ft/s}}}$$