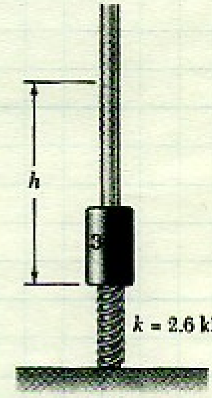
Set #8 – Conservation of Energy

1. A 3-kg collar can slide without friction on a vertical rod and is resting in equilibrium on a spring. It is pushed down, compressing the spring 150mm, and released. Knowing that the spring constant k = 2.6kN/m, determine

a) the maximum height h reached by the collar above its equilibrium position,

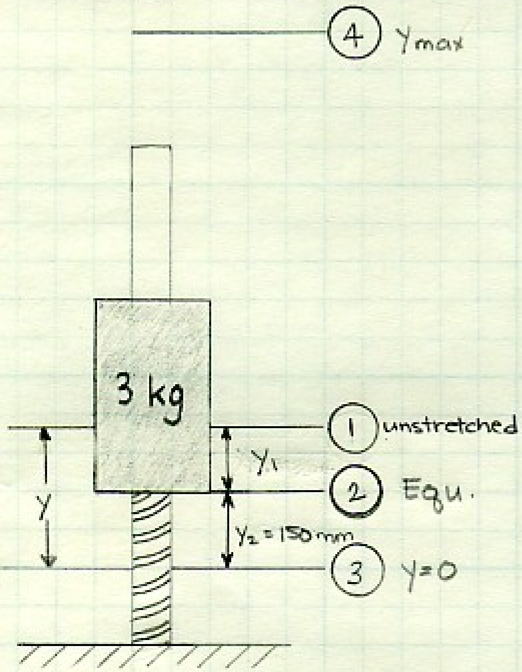
b) the maximum velocity of the collar.



Given: $m = 3\,kg$
Frictionless
resting on spring
compressed $150\,mm = .150\,m$
$K = 2.6\,kN/m = 2600\,N/m$

Find: (a) max. height, Y_{max}, above equilibrium position.
(b) max. velocity, V_{max}.

Solution:



1 → This is the position of the unstretched spring

2 → Because the collar is resting on the spring, it compresses the spring to this equilibrium position.

3 → The collar is pushed down 150 mm.

4 → The maximum height that the collar reaches once it is released.

First, we must find the distance x. (1 to 2)

$$F = k y_1$$

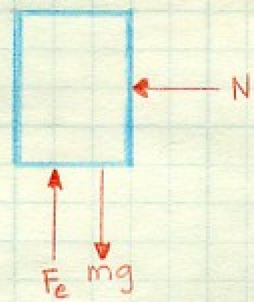
$$mg = k y_1 \longrightarrow y_1 = \frac{mg}{k}$$

$$= \frac{(3\,kg)(9.81\,m/s^2)}{2600\,N/m}$$

$$= .01132\,m$$

Set #8

1. continued.



Note: This FBD only correponds to the points in time when the collar remains in contact with the spring.
Once the spring reaches its undeflected length, the collar leaves the spring.

(a.) $T_3 + U_{1\to 2} = T_4$ —①

$$\left.\begin{aligned} U_{1\to 2} &= V_{g_1} - V_{g_2} \\ U_{1\to 2} &= V_{e_1} - V_{e_2} \end{aligned}\right\} \text{—②}$$

Plugging ② into ①, we obtain the following:

$$\rightarrow T_3 + V_{g_3} + V_{e_3} = T_4 + V_{g_4} + V_{e_4}$$
$$0 + 0 + \tfrac{1}{2} k y^2 = 0 + mgy_4.$$

$$y_4 = \frac{ky^2}{2mg}$$
$$= \frac{2600\,(.150 + .01132)^2}{2(3)(9.81)}$$
$$= 1.15 \text{ m}$$

So, $y_{max} = 1.15\text{ m} - .15\text{ m}$
$= \underline{\underline{1.00 \text{ m}}}$ above equilibrium position

Set #8

1. continued.

(b) V_{max} occurs at equilibrium position because $\frac{dv}{dt} = 0$ when $a = 0$

and $a = 0$ when $F_{net} = 0$

$$T_3 + V_{g_3} + V_{e_3} = T_2 + V_{g_2} + V_{e_2}$$

$$0 + 0 + \tfrac{1}{2} k y^2 = \tfrac{1}{2} m v_{max}^2 + m g y_2 + \tfrac{1}{2} k y_1^2$$

$$(2600)(.15 + .01132)^2 = (3) v_{max}^2 + (2)(3)(9.81)(.15) + (2600)(.01132)^2$$

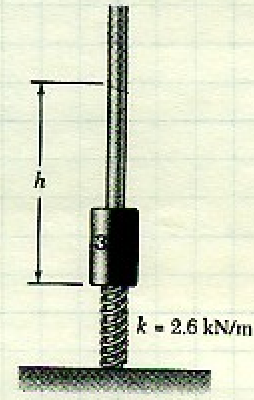
$$\underline{\underline{v_{max} = 4.42 \text{ m/s}}}$$

Set #8 – Conservation of Energy

2. A 3-kg collar can slide without friction on a vertical rod and is held so that it just touches an undeformed spring. Determine the maximum deflection of the spring

a) if the collar is slowly released until it reaches an equilibrium position,

b) if the collar is suddenly released.



Given: $m = 3\,kg$ $\quad$ $k = 2.6\ kN/m$

Frictionless

Collar just touches undeformed spring.

Find: Determine the maximum deflection of the spring if.

(a.) the collar is slowly released.

(b.) the collar is suddenly released.

Solution:

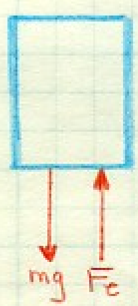
(a.) $F = kx \longrightarrow x = \frac{F}{k}$

$$= \frac{mg}{k}$$

$$= \frac{(3kg)(9.81\ m/s)}{2600\ N/m}$$

$$= \underline{\underline{11.32\ mm}}$$

(b.)



$$T_1 + V_{g_1} + V_{e_1} = T_2 + V_{g_2} + V_{e_2}$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}ky_2^2 + mgy_2$$

$$y_2 = -\frac{2mg}{k}$$

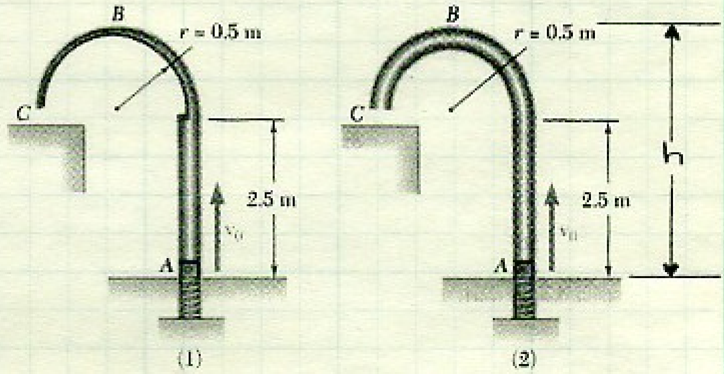
$$= \frac{-2(3)(9.81)}{2600}$$

$$= \underline{\underline{22.6\ mm}}$$

Set #8 – Conservation of Energy

3. A 200-g package is projected upward with a velocity $\mathbf{v}_0$ by a spring at A; it moves around a frictionless loop and is deposited at C. For each of the two loops shown, determine

a) the smallest velocity $\mathbf{v}_0$ for which the package will reach C,
b) the corresponding force exerted by the package on the loop just before the package leaves the loop at C.

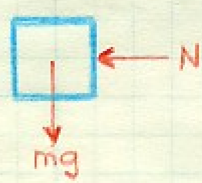


Given: $m = 200\text{ g} = .2\text{ kg}$
$V_0 \uparrow$
Frictionless

Find: For each loop, determine
(a.) $V_{o\,min}$ to reach C without bottom of track
$V_{o\,min}$ to reach C with bottom.
(b.) Force exerted on loop at C (without bottom).
Force exerted on loop at C (with bottom.)

Solution:

(a.)



To get V_o:

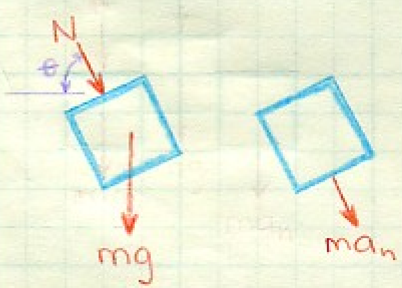
$$T_A + V_A = T_B + V_B$$
$$\tfrac{1}{2}mv_o^2 + 0 = \tfrac{1}{2}mv_B^2 + mgh$$

(Note: Refer to figure for h.)

$$v_o^2 = V_B^2 + 2mgh \quad \text{—①}$$

In order to obtain V_o, we need $V_{B\,min}$.

Case #1 (no bottom)



We need finite N at all points between B and C.

(Consider B^+ any point between B & C)

$$\Sigma F_n = ma_n$$
$$N + mg\sin\theta = ma_n$$
$$N + mg\sin\theta = m\frac{v_{B^+}^2}{r}$$
$$V_{B^+}^2 = \frac{rN}{m} + rg\sin\theta$$

Set #8

3. continued.

For minimum v_o, we need minimum v_B^2.
For minimum v_B^2, we need $N = 0$ and $\theta = 90°$

$$v_B^2 = \cancelto{0}{\frac{Nr}{m}} + rg \sin 90°$$

$$v_B^2 = rg \quad — ②$$

Substituting ② into ①,

$$v_o^2 = rg + 2gh$$

$$\underline{\underline{v_o = 7.98 \text{ m/s}}}$$

<u>Case #2</u> (with bottom)

We need finite velocity at B.

For minimum v_o, we need minimum v_B ($v_{B\min} = 0$).

$$v_o^2 = v_B^2 + 2gh$$

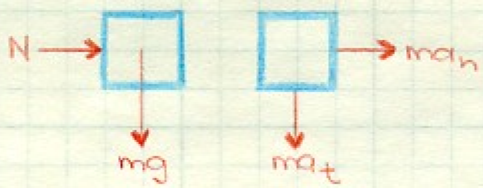
$$= 2gh$$

$$\underline{\underline{v_o = 7.67 \text{ m/s}}}$$

Set #8

3. continued

(b) Force exerted on loop at C.



$$\Sigma F_n = ma_n$$
$$N = m\frac{v_c^2}{r} \quad \text{—③}$$

We need v_c:
$$T_A + V_A = T_c + V_c$$
$$\tfrac{1}{2}\cancel{m}v_0^2 + 0 = \tfrac{1}{2}\cancel{m}v_c^2 + \cancel{m}g(2.5)$$

$$v_c^2 = v_0^2 - 5g \quad \text{—④}$$

<u>Case #1</u> (no bottom)

Substituting ① into ④

$$v_c^2 = v_0^2 - 5g$$
$$= (rg + 2gh) - 5g$$
$$= rg + \cancel{2g(2.5)} - \cancel{5g}$$
$$v_c^2 = rg \quad \text{—⑤}$$

Now, we can solve for N (equation ③).
Plug in ⑤.

$$N = m\frac{v_c^2}{r}$$
$$= \frac{m(1.5g)}{r}$$
$$\underline{\underline{N = 5.89\ N}}$$

<u>Case #2</u> (with bottom)

Since v_B in ① is 0, $v_0^2 = 2gh$.
Substituting into ④.

$$v_c^2 = 2gh - 5g$$
$$= 2(3)g - 5g$$
$$= g$$

$$N = \frac{mg}{r}$$
$$\underline{\underline{= 3.92\ N}}$$