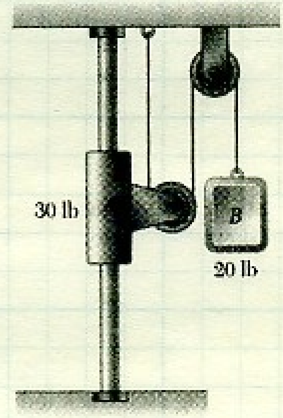
Set #10 – Impulse and Momentum

1. The system shown is released from rest. Determine the time it takes for the velocity of A to reach 2 ft/s. Neglect friction and the mass of the pulleys.

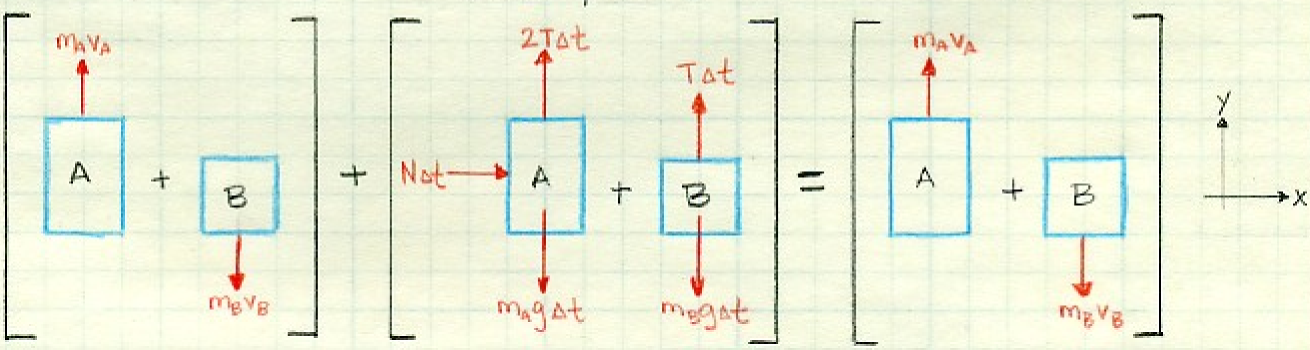


Given: $V_{A_o} = V_{B_o} = 0$
Frictionless
Massless Pulleys

Find: t for $V_A = 2\ \text{ft/s}$

Solution: Principle of Impulse & Momentum.

$$mV_1 + \Sigma Imp_{1\rightarrow 2} = mV_2$$



$+\uparrow y$ components:

$$\text{For A: } [0] + [2T\Delta t - m_A g\Delta t] = [m_A v_A] \quad (1)$$
$$\text{For B: } [0] + [T\Delta t - m_B g\Delta t] = [-m_B v_B] \quad (2)$$

Adding (1) and (2); Multiplying (2) by -2.

$$\cancel{2T\Delta t} - m_A g\Delta t = m_A v_A$$
$$\cancel{-2T\Delta t} + 2m_B g\Delta t = 2m_B v_B$$

$$2m_B g\Delta t - m_A g\Delta t = m_A v_A + 2m_B v_B$$
$$g\Delta t(2m_B - m_A) = m_A v_A + 2m_B v_B$$

$$\Delta t = \frac{m_A v_A + 2m_B v_B}{g(2m_B - m_A)}$$

Set #10

1. continued.

$$\Delta t = \frac{m_A V_A + 2 m_B V_B}{g(2m_B - m_A)}$$

$$\Delta t = \frac{m_A V_A + 2 m_B (2V_A)}{g(2m_B - m_A)}$$

$$\Delta t = \frac{m_A V_A + 4 m_B V_A}{g(2m_B - m_A)}$$

$$\Delta t = \frac{V_A}{g} \frac{(m_A + 4 m_B)}{(2m_B - m_A)}$$

$$\Delta t = \frac{2\,ft/s}{32.2\,ft/s^2} \frac{\left(\frac{30\ lbf}{32.2\ ft/s^2} + \frac{4(20\ lbf)}{32.2\ ft/s^2}\right)}{\left(\frac{2(20\ lb)}{32.2\ ft/s^2} - \frac{30\ lb}{32.2\ ft/s^2}\right)}$$

$$\underline{\underline{\Delta t = 0.683\ sec}}$$

Set #10 – Impulse and Momentum

2. A mother and her child are skiing together, with the mother holding the end of a rope tied to the child's waist. They are moving at a speed of 7.2 km/h on a flat portion of the ski trail when the mother observes that they are approaching a steep descent. She decides to pull on the rope to decrease the child's speed. Knowing that this maneuver causes the child's speed to be cut in half in 3 s and neglecting friction, determine

a) the mother's speed at the end of the 3-s interval,

b) the average value of the tension in the rope during that time interval.

Given: $V_{c_0} = V_{m_0} = 7.2 \ \mathrm{km/hr} = 2 \ \mathrm{m/s}$

After $t = 3$ sec,

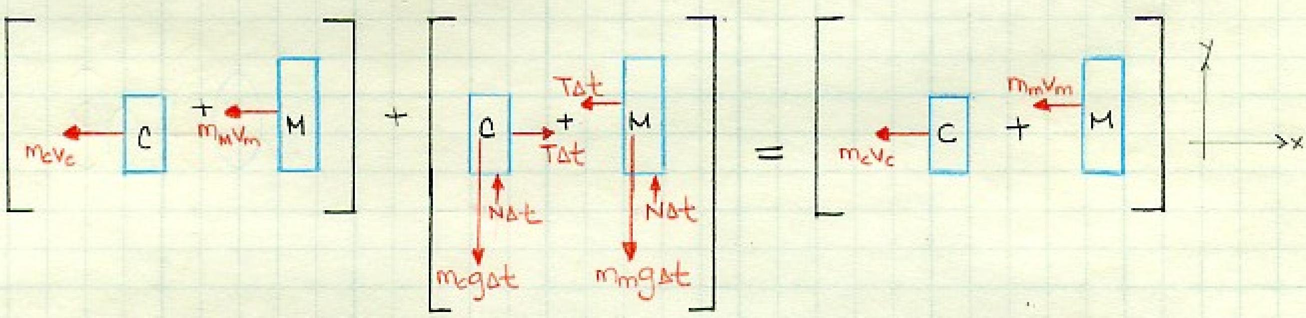
$V_c = \frac{1}{2} V_{c_0} = 3.6 \ \mathrm{km/hr} = 1 \ \mathrm{m/s}$

Frictionless.

Find: (a) V_m

(b) T_{AVE}

Solution: Use Principle of Impulse and Momentum.



$$mv_1 \quad + \quad Imp_{1\to 2} \quad = \quad mv_2$$

Note: It is not necessary to draw internal forces (i.e. gravity, etc) unless, of course, they are needed.

$\xrightarrow{+}$ x-direction:

For child: $\quad -m_c v_{c_0} + T\Delta t = -m_c v_{c_f} \quad$ — (1)

For Mother: $\quad -m_m v_{m_0} - T\Delta t = -m_m v_{m_f} \quad$ — (2)

Set #10

2. Continued

$\xrightarrow{+}$ x-direction:

For child: $-m_c v_{c_0} + \cancel{T}\Delta t = -m_c v_{c_f}$

For mother: $-m_m v_{m_0} - \cancel{T}\Delta t = -m_m v_{m_f}$

(a) $$-m_c v_{c_0} - m_m v_{m_0} = -m_c v_{c_f} - m_m v_{m_f}$$

$$v_0 (m_c + m_m) = m_c v_{c_f} + m_m v_{m_f}$$

$$v_{m_f} = \frac{v_0 (m_c + m_m) - m_c v_{c_f}}{m_m}$$

$$v_{m_f} = \frac{(2\ m/s)(20\ kg + 55\ kg) - (20\ kg)(1\ m/s)}{(55\ kg)}$$

$$v_{m_f} = 2.36\ m/s$$

$$v_{m_f} = 8.51\ km/hr$$

(b) To solve for T_{AVE}, either equation ① or ② can be used. Simply solve for T.

$$-m_c v_{c_0} + T\Delta t = -m_c v_{c_f}$$

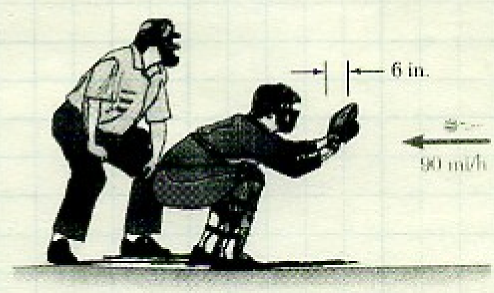
$$T = \frac{m_c (v_{c_0} - v_{c_f})}{\Delta t}$$

$$= \frac{(20\ kg)(2\ m/s - 1\ m/s)}{3\ sec}$$

$$= \underline{\underline{6.67\ N\ (T)}}$$

Set #10 – Impulse and Momentum

3. A baseball player catching a ball can soften the impact by pulling her hand back. Assuming that a 5-oz ball reaches her glove at 90 mph and that the player pulls her hand back during the impact at an average speed of 30 ft/s over a distance of 6 inches, bringing the ball to a stop, determine the average impulsive force exerted on the player's hand.



Given: $V_{B_o} = 90 \text{ mph} = 132 \text{ ft/s}$

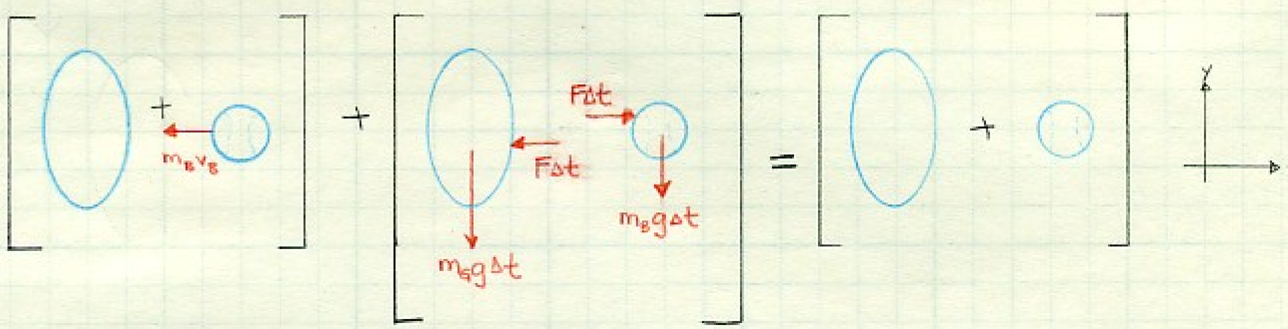
$V_G = 30 \text{ ft/s}$

$d = 6 \text{ in.} = .5 \text{ ft.}$

$V_{B_f} = 0$

Find: F_{AVE}

Solution:



$$mv_1 + Imp_{1\to 2} = mv_2$$

$\xrightarrow{+}$ x-direction:

For ball: $-m_B V_{B_o} + F\Delta t = 0$

Note: We are not directly given Δt. However, we are given information about the glove.

$$V_G = \frac{\Delta X}{\Delta t} \longrightarrow \Delta t = \frac{\Delta X}{V_G}$$

$$= \frac{.5 \text{ ft}}{30 \text{ ft/s}}$$

$$= .0167 \text{ sec}$$

Set #10

3. continued

$$-m_B V_{B_0} + F\Delta t = 0$$

$$F = \frac{m_B V_{B_0}}{\Delta t}$$

$$F = \frac{\left(\dfrac{\frac{5\,oz}{16\,oz}}{32.2\ ft/s^2}\right)(132\ ft/s)}{.0167\ sec}$$

$$\underline{\underline{F = 76.71\ lb_f}}$$