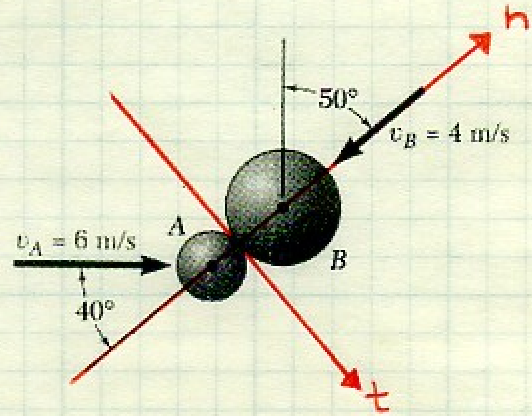
Set #11 – Impact

1. A 600-g ball A is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball B which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.

Given: $m_A = 600g = .6\ kg$
$v_A = 6\ m/s$
$m_B = 1\ kg$
$v_B = 4\ m/s$
$e = 0.8$
Frictionless



Find: v_A', v_B'

Solution:

- No external forces
  → Momentum is conserved

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad (1)$$

- No friction
  → Momentum for each mass is conserved in t-dir.

$$m_A v_{At} = m_A v_{At}' \qquad m_B v_{Bt} = m_B v_{Bt}'$$

$$\rightarrow v_{At}' = v_{At} \sin 40^\circ = (6\ m/s)\sin 40^\circ = 3.8567\ m/s \qquad \rightarrow v_{Bt}' = 0$$

- Using the equation for relative velocities before and after the impact in the normal direction only.

$$v_{Bn}' - v_{An}' = e\,(v_{An} - v_{Bn})$$

$$\rightarrow v_{Bn}' = 0.8\,(6\ m/s \cos 40^\circ - (-4\ m/s)) + v_{An}'$$

$$v_{Bn}' = 6.877\ m/s + v_{An}' \quad (2)$$

Set #11

1. continued

- Using equation 1, we get

$$m_A v_{An} + m_B v_{Bn} = m_A v'_{An} + m_B v'_{Bn}$$

$$(.6\ \text{kg})(6\ \text{m/s} \cos 40^\circ) + (1\ \text{kg})(-4\ \text{m/s}) = (.6\ \text{kg})\, v'_{An} + (1\ \text{kg})(6.877\ \text{m/s} + v'_{An})$$

$$-1.2422\ \text{kg}\cdot\text{m/s} = (.6\ \text{kg})\, v'_{An} + 6.877\ \text{kg}\cdot\text{m/s} + (1\ \text{kg})\, v'_{An}$$

$$-8.1192\ \text{kg}\cdot\text{m/s} = v'_{An}\,(.6\ \text{kg} + 1\ \text{kg})$$

$$\rightarrow v'_{An} = -5.0745\ \text{m/s}$$

v'_A : $v'_{At} = 3.8567$ m/s
$v'_{An} = -5.0745$ m/s

$$\longrightarrow \underline{\underline{v_A = 6.37\ \text{m/s} \quad 37.8^\circ}}$$

v'_B : $v'_{Bt} = 0$
$v'_{Bn} = 6.877$ m/s $+ (-5.0745$ m/s$)$ — 2
$= 1.8025$ m/s

$$\longrightarrow \underline{\underline{v_B = 1.8025\ \text{m/s} \quad 40^\circ}}$$

Set #11 – Impact

2. A 3-lb sphere A strikes the frictionless inclined surface of a 9-lb wedge B at a 90° angle with a velocity of magnitude 12ft/s. The wedge can roll freely on the ground and is initially at rest. Knowing that the coefficient of restitution between the wedge and the sphere is 0.50 and that the inclined surface of the wedge forms an angle θ = 40° with the horizontal, determine

a) the velocities of the sphere and of the wedge immediately after impact,

b) the energy lost due to the impact.

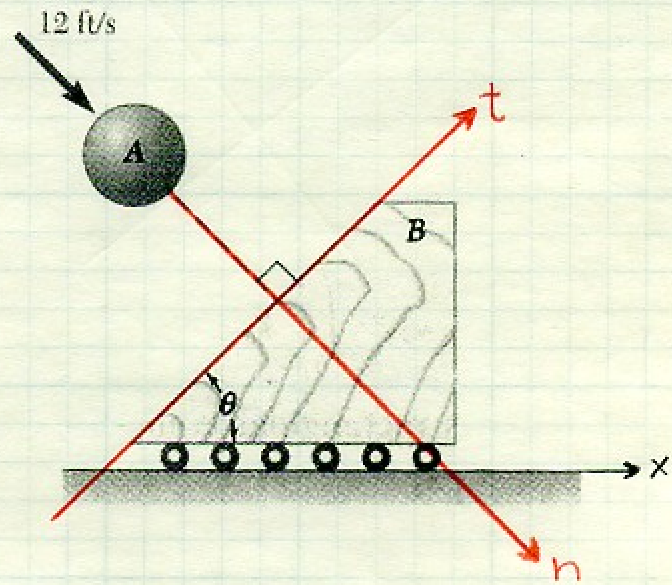
Given: $v_A = 12\ ft/s$

$v_B = 0$

$e = 0.50$

$\theta = 40°$

Frictionless



Find: (a) v'_A , v'_B

(b) Energy loss.

Solution:

Assumptions:

- Impulses do to weight can be neglected
- Wedge B stays on the ground ($v_{By} = 0$)

• No external forces in the x-direction

→ Momentum in x-dir. conserved ($L_{1x} = L_{2x}$)

$$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$$

$$m_A v_{Ax} + 0 = m_A v'_{Ax} + m_B v'_{Bx}$$

$$\rightarrow v'_{Bx} = \frac{m_A v_{Ax} - m_A v'_{Ax}}{m_B}$$

$$= \frac{(3lb)(12\ ft/s \cos 50°) - (3lb)\, v'_{Ax}}{(9lb)}$$

$$v'_{Bx} = 4\ ft/s \cos 50° - \frac{v'_{Ax}}{3} \quad \text{—(1)}$$

Note: If there is confusion as to how v_{Ax} was obtained, the figure on the following page can help. The same technique was applied.

• No friction

→ Momentum in the t-direction is conserved for A.

$$m_A v_{At} = m_A v'_{At}$$

$$0 = m_A v'_{At}$$

$$v'_A = 0 \quad \text{—(2)}$$

Set #11

2. continued

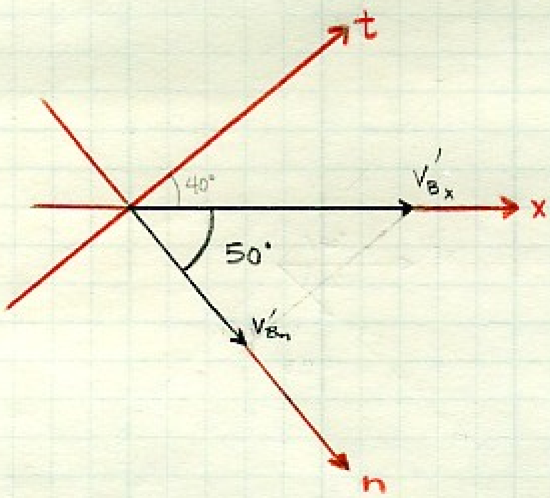
- Using the equation for relative velocities before and after the impact in the normal direction <u>only</u>.

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn}) \quad (3)$$

- We have followed the usual steps in solving problems such as these. However, because of the addition x-axis, we consequently have additional work to perform.
  Now, we have to find the term v_{Bn} in order to be able to use equation (3).

Be careful; this can get a bit tricky!

We are given the following.



From the figure,

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 50° = \frac{v'_{Bn}}{v'_{Bx}}$$

$$\rightarrow v'_{Bn} = v'_{Bx} \cos 50° \quad (4)$$

We can now substitute (1) into (4).

$$v'_{Bn} = \left(4\,ft/s^2 \cos 50° - \frac{v'_{Ax}}{3}\right)\cos 50°$$

$$= 4\,ft/s^2 \cos^2 50° - \frac{v'_{Ax}\cos 50°}{3}$$

$$= 4\,ft/s^2 \cos^2 50° - \frac{(v'_{An}\cos 50°)\cos 50°}{3}$$

$$= 4\,ft/s^2 \cos^2 50° - \frac{v'_{An}\cos^2 50°}{3} \quad (5)$$

Set #11

2. continued

Plugging ⑤ into ③, we can solve for V'_{An}

$$\left(4\cos^2 50° - \frac{V'_{An}\cos^2 50°}{3}\right) - V'_{An} = e\,(V_{An} - \cancel{V_{Bn}}^{\,0})$$

$$-\frac{V'_{An}\cos^2 50°}{3} - V'_{An} = 4.3473 \text{ ft/s}$$

$$V'_{An}\left(\frac{\cos^2 50°}{3} + 1\right) = -4.3473 \text{ ft/s}$$

$$V'_{An} = -3.82 \text{ ft/s}$$

$$\Longrightarrow \underline{\underline{V'_A = 3.82 \text{ ft/s} \quad 50°}}$$

Using equation ①,

$$V'_{Bx} = 4\cos 50° - \frac{V'_{Ax}}{3}$$

$$= 4\cos 50° - \frac{(V'_{An}\cos 50°)}{3}$$

$$= 4\cos 50° - \frac{(-3.82\cos 50°)}{3}$$

$$\underline{\underline{V'_{Bx} = 3.39 \text{ ft/s} \longrightarrow}}$$