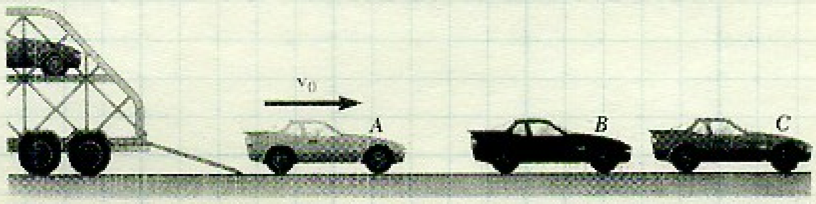
Set #12 – Systems of Particles

1. Three identical cars are being unloaded from an automobile carrier. Cars B and C have just been unloaded and are at rest with their brakes off when car A leaves the unloading ramp with a velocity of 2.00 m/s and hits car B, which in turn hits car C. Car A then again hits car B. Knowing that the velocity of car A is 0.400 m/s after its first collision with car B and 0.336 m/s after its second collision with car B and that the velocity of car C is 1.280 m/s after it has been hit by car B, determine

a) the velocity of car B after each of the three collisions,
b) the coefficient of restitution between any two of the three cars.



Given: $m = m_A = m_B = m_C$
$v_B = v_C = 0$
$v_A = 2.00\ m/s$
$v_A' = 0.400\ m/s$
$v_A'' = 0.336\ m/s$
$v_C' = 1.280\ m/s$

Note: $'$ refers to immediately after first collision
$''$ refers to immediately after second collision
$'''$ refers to immediately after third collision.

Find: (a.) v_B', v_B'', v_B'''
(b.) e

Solution:
(a)
• Car A into B the first time.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$v_A + v_B = v_A' + v_B'$$
$$\rightarrow v_B' = v_A + v_B - v_A'$$
$$v_B' = 2.00\ m/s + 0 - 0.400\ m/s$$
$$\underline{\underline{v_B' = 1.60\ m/s}}$$

Set #12

1. continued.

- B into C

$$\cancel{m_B} v_B' + \cancel{m_c} v_c = \cancel{m_B} v_B'' + \cancel{m_c} v_c'$$

$$v_B' + v_c = v_B'' + v_c'$$
$$\rightarrow v_B'' = v_B' + v_c - v_c'$$
$$v_B'' = 1.60\,m/s + 0 - 1.280\,m/s$$
$$\underline{\underline{v_B'' = .320\,m/s}}$$

- A into B the second time

$$\cancel{m_A} v_A' + \cancel{m_B} v_B'' = \cancel{m_A} v_A'' + \cancel{m_B} v_B'''$$

$$v_A' + v_B'' = v_A'' + v_B'''$$
$$\rightarrow v_B''' = v_A' + v_B'' - v_A''$$
$$v_B''' = 0.400\,m/s + .320\,m/s - 0.336\,m/s$$
$$\underline{\underline{v_B''' = .384\,m/s}}$$

(b) Coefficient of restitution

$$v_B' - v_A' = e(v_A - v_B)$$
$$\rightarrow e = \frac{v_B' - v_A'}{v_A - v_B}$$
$$= \frac{1.60\,m/s - 0.400\,m/s}{2.00\,m/s - 0}$$
$$\underline{\underline{e = 0.6}}$$

Set #12 – Systems of Particles

2. A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 5$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v_A} = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, $\mathbf{v_B} = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$, $\mathbf{v_C} = 2\mathbf{i} + -6\mathbf{j} + -4\mathbf{k}$.

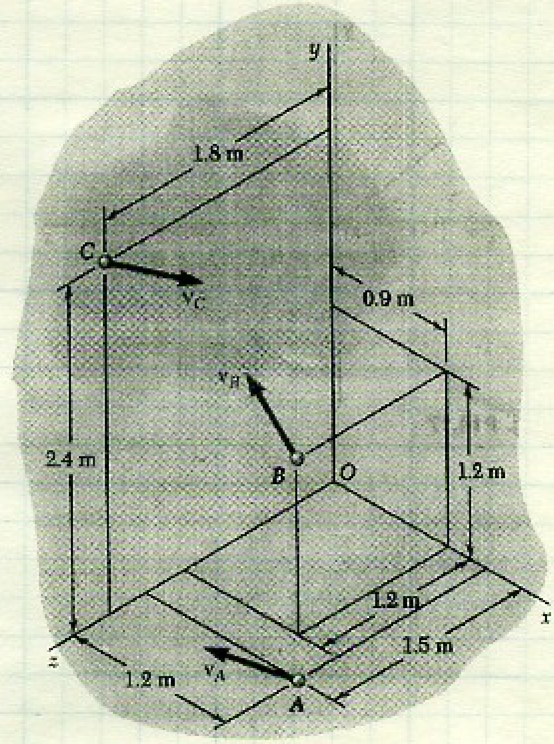
a) Determine the angular momentum $\mathbf{H_o}$ of the system about O.

b) Determine the position vector $\bar{r}$ of the mass center G of the system,

c) Determine the linear momentum $m\bar{v}$ of the system,

d) Determine the angular momentum $\mathbf{H_G}$ of the system about G.

e) Verify this equation: $\mathbf{H_o} = \bar{r} \times m\bar{v} + \mathbf{H_G}$



The vectors $\bar{r}$ and $\bar{v}$ define, respectively, the position and velocity of the mass center G of the system of particles relative to the Newtonian frame of reference Oxyz, and m represents the total mass of the system.

Given: $m_A = 3\,kg$, $m_B = 4\,kg$, $m_c = 5\,kg$

$\vec{V}_A = (-4\hat{i} + 4\hat{j} + 6\hat{k})$ m/s

$\vec{V}_B = (-6\hat{i} + 8\hat{j} + 4\hat{k})$ m/s

$\vec{V}_c = (2\hat{i} - 6\hat{j} - 4\hat{k})$ m/s

$\vec{r}_A = (1.2\hat{i} + 0\hat{j} + 1.5\hat{k})$ m

$\vec{r}_B = (0.9\hat{i} + 1.2\hat{j} + 1.2\hat{k})$ m

$\vec{r}_c = (0\hat{i} + 2.4\hat{j} + 1.8\hat{k})$ m

Find: (a) $\vec{H}_o$

(b) $\vec{\bar{r}}$

(c) $\vec{L} = m\vec{\bar{v}}$

(d) $\vec{H}_G$

(e) Verify $\vec{H}_o = \vec{\bar{r}} \times m\vec{\bar{v}} + \vec{H}_G$

Solution:

(a) $\vec{H}_o = \sum \vec{r}_i \times m_i v_i$

$$= 3\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & 0 & 1.5 \\ -4 & 4 & 6 \end{vmatrix} + 4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.9 & 1.2 & 1.2 \\ -6 & 8 & 4 \end{vmatrix} + 5\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2.4 & 1.8 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= 3(-6\hat{i} - 13.2\hat{j} + 4.8\hat{k}) + 4(-4.8\hat{i} - 10.8\hat{j} + 14.4\hat{k}) + 5(1.2\hat{i} + 3.6\hat{j} - 4.8\hat{k})$$

$$\vec{H}_o = -31.2\hat{i} - 64.8\hat{j} + 48\hat{k} \quad kg\,m^2/s$$

Set #12

2. continued

(b.) $m\vec{\bar{r}} = \Sigma m_i \vec{r}_i \longrightarrow \vec{\bar{r}} = \dfrac{\Sigma m_i r_i}{m_{TOTAL}}$

$$\vec{\bar{r}} = \frac{3(1.2\hat{i} + 0\hat{j} + 1.5\hat{k}) + 4(.9\hat{i} + 1.2\hat{j} + 1.2\hat{k}) + 5(0\hat{i} + 2.4\hat{j} + 1.8\hat{k})}{(3+4+5)}$$

$$\underline{\underline{\vec{\bar{r}} = (.6\hat{i} + 1.4\hat{j} + 1.525\hat{k})\ m/s}}$$

(c) $\vec{L} = m\vec{\bar{v}} = \Sigma m_i \vec{v}_i$

$$\vec{L} = 3(-4\hat{i} + 4\hat{j} + 6\hat{k}) + 4(-6\hat{i} + 8\hat{j} + 4\hat{k}) + 5(2\hat{i} - 6\hat{j} - 4\hat{k})$$
$$= \underline{\underline{(-26\hat{i} + 14\hat{j} + 14\hat{k})\ kg \cdot m/s}}$$

(d) $\vec{H}_G = \Sigma \vec{r}_i{}' \times m_i \vec{v}_i$

$$\vec{r}_i{}' = \vec{r} - \vec{\bar{r}}$$
$$r_A' = (1.2\hat{i} + 0\hat{j} + 1.5\hat{k}) - (.6\hat{i} + 1.4\hat{j} + 1.525\hat{k})$$
$$r_A' = (.6\hat{i} - 1.4\hat{j} - .025\hat{k})\ m$$
$$r_B' = (.3\hat{i} - .2\hat{j} - .325\hat{k})\ m$$
$$r_C' = (-.6\hat{i} + \hat{j} + .275\hat{k})\ m$$

$$\vec{H}_G = 3\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .6 & -1.4 & -.025 \\ -4 & 4 & 6 \end{vmatrix} + 4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .3 & -.2 & -.325 \\ -6 & 8 & 4 \end{vmatrix} + 5\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -.6 & 1 & .275 \\ 2 & -6 & -4 \end{vmatrix}$$

$$\underline{\underline{\vec{H}_G = (-29.45\hat{i} - 16.75\hat{j} + 3.2\hat{k})\ kg \cdot m^2/s}}$$

(e.) $\vec{H}_O = \vec{\bar{r}} \times m\vec{\bar{v}} + H_G$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ .6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix} + (-29.45\hat{i} - 16.75\hat{j} + 3.2\hat{k})$$

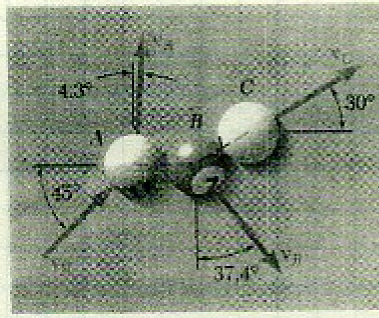
$$= (-1.75\hat{i} - 48.05\hat{j} + 44.8\hat{k}) + (-29.45\hat{i} - 16.75\hat{j} + 3.2\hat{k})$$

$$\underline{\underline{\vec{H}_O = -31.2\hat{i} - 64.8\hat{j} + 48\hat{k}}}$$

Set #12 – Systems of Particles

3. In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and that $v_0 = 12$ ft/s and $v_C = 6.29$ ft/s, determine the magnitude of the velocity of

a) ball A,
b) ball B.



Given: $m_A = m_B = m_C$

$V_A = 12$ ft/s

$V_B = 0$

$V_C = 0$

$V_C' = 6.29$ ft/s

Find: (a) V_A'

(b) V_B'

Solution:

There are no external force

$\rightarrow$ Momentum is conserved.

$$\cancel{m_A} V_A + \cancel{m_B} V_B + \cancel{m_C} V_C = \cancel{m_A} V_A' + \cancel{m_B} V_B' + \cancel{m_C} V_C'$$

$$V_A + V_B + V_C = V_A' + V_B' + V_C'$$

$\xrightarrow{x}$ $12 \cos 45° + 0 + 0 = V_A' \sin 4.3° + V_B' \sin 37.4° + 6.29 \cos 30°$

$$V_A' \sin 4.3° + V_B' \sin 37.4° = 3.038 \text{ ft/s} \quad (1)$$

$y\uparrow$ $12 \sin 45° + 0 + 0 = V_A' \cos 4.3° - V_B' \cos 37.4° + 6.29 \sin 30°$

$$V_A' \cos 4.3° - V_B' \cos 37.4° = 5.3403 \text{ ft/s} \quad (2)$$

- Adding (1) and (2),

$$(V_A' \sin 4.3° + V_B' \sin 37.4° = 3.038) \cos 37.4°$$
$$(V_A' \cos 4.3° - V_B' \cos 37.4° = 5.3403) \sin 37.4°$$

$$V_A' \cos 37.4° \sin 4.3° + V_B' \cancel{\cos 37.4° \sin 37.4°} = 3.038 \cos 37.4°$$
$$V_A' \cos 4.3° \sin 37.4° - V_B' \cancel{\cos 37.4° \sin 37.4°} = 5.3403 \sin 37.4°$$

$$V_A' \cos 37.4° \sin 4.3° + V_A' \cos 4.3° \sin 37.4° = 3.038 \cos 37.3° + 5.3403 \sin 37.4°$$

$$V_A' (.66523) = 5.6602$$

$$\underline{\underline{V_A' = 8.51 \text{ ft/s}}}$$

- Rearranging (1),

$$V_B' = \frac{3.038 - (8.51) \sin 4.3}{\sin 37.4}$$

$$\underline{\underline{V_B' = 3.95 \text{ ft/s}}}$$