Set #14 – Motion: Translation & Rotation

1. The angular acceleration of a shaft is defined by the relation $\alpha = -0.25\ \omega$, where α is expressed in rad/s^2 and ω in rad/s. Knowing that at $t = 0$ the angular velocity of the shaft is 20 rad/s, determine

a) the number of revolutions the shaft will execute before coming to rest,

b) the time required for the shaft to come to rest,

c) the time required for the angular velocity of the shaft to be reduced to 1% of its initial value.

Given: $\alpha = -0.25\ \omega$

$\omega_o = 20\ \text{rad/s}$

Find: (a) number of revolutions to come to rest

(b) time to come to rest

(c) time for ω to reduce 1% of initial value.

Solution:

(a)

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha\, dt = d\omega$$

$$(-0.25\omega)\, dt = d\omega$$

$$-0.25\left(\frac{d\theta}{dt}\right) dt = d\omega$$

$$-0.25\, d\theta = d\omega$$

$$\int_0^\theta -0.25\, d\theta = \int_{\omega_o}^0 d\omega$$

$$-0.25\int_0^\theta d\theta = \int_{\omega_o}^0 d\omega$$

$$-0.25\left[\theta\right]_0^\theta = \left[\omega\right]_{\omega_o}^0$$

$$-0.25(\theta - 0) = 0 - \omega_o$$

$$-0.25\theta = -\omega_o$$

$$-0.25\theta = -(20\ \text{rad/s})$$

$$\theta = \frac{-20\ \text{rad/s}}{-0.25}$$

$$= 80\ \text{rad/s}$$

$$= \underline{\underline{12.73\ \text{revolutions}}}$$

Set #4

1. continued

(b.)

$$\frac{d\omega}{dt} = -0.25\,\omega$$

$$\frac{d\omega}{\omega} = -0.25\,dt$$

$$\int_{\omega_0}^{0} \frac{d\omega}{\omega} = \int_0^t -0.25\,dt$$

$$\int_{\omega_0}^{0} \frac{d\omega}{\omega} = -0.25 \int_0^t dt$$

$$[\ln \omega]_{\omega_0}^{0} = -0.25\,[t]_0^t$$

$$\ln 0 - \ln \omega_0 = -0.25(t-0)$$

$$\ln \omega_0 - \ln 0 = 0.25t$$

$$t = \frac{\ln \omega_0 - \ln 0}{0.25}$$

$$= \underline{\underline{\text{infinite}}}$$

(c.)

$$\int_{\omega_0}^{.01\omega_0} \frac{d\omega}{\omega} = \int_0^t -0.25\,dt$$

$$[\ln \omega]_{\omega_0}^{.01\omega_0} = -0.25\,[t]_0^t$$

$$\ln .01\omega_0 - \ln \omega_0 = -0.25t$$

$$\ln \frac{.01\,\omega_0}{\omega_0} = -0.25t$$

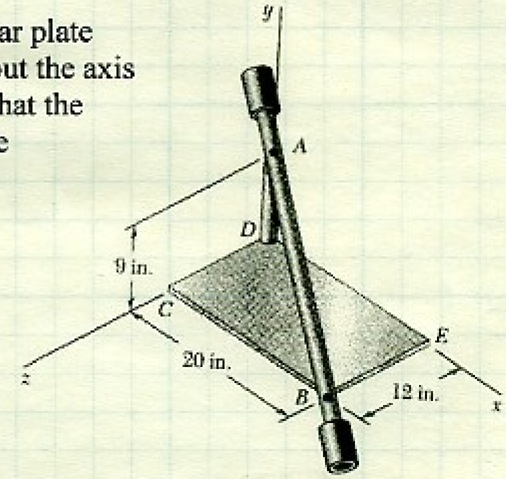
$$\ln .01 = -0.25t$$

$$t = \frac{\ln .01}{-0.25}$$

$$= \underline{\underline{18.42 \text{ sec}}}$$

Set #14 – Motion: Translation & Rotation

2. The assembly shown consists of two rods and a rectangular plate BCDE which are welded together. The assembly rotates about the axis AB with a constant angular velocity of 7.5 rad/s. Knowing that the rotation is counterclockwise as viewed from B, determine the velocity and acceleration of corner E.



Given: $\omega_{AB} = 7.5 \text{ rad/s}$ (constant)

$\circlearrowleft$ viewed from B.

$\vec{\alpha} = \vec{0}$

Find: $\vec{v}_E$

$\vec{a}_E$

Solution:

In order to find $\vec{\omega}$, we need to find $\hat{\lambda}_{AB}$ $\quad (\vec{\omega} = \omega\hat{\lambda})$

$$\hat{\lambda}_{AB} = \frac{(20-0)\hat{i} + (0-9)\hat{j} + (12-0)\hat{k}}{\sqrt{(20-0)^2 + (0-9)^2 + (12-0)^2}}$$

$$= \frac{20\hat{i} - 9\hat{j} + 12\hat{k}}{25}$$

$$= \frac{4}{5}\hat{i} - \frac{9}{25}\hat{j} + \frac{12}{25}\hat{k} \text{ in}$$

$$\vec{\omega}_{AB} = \omega\hat{\lambda}_{AB}$$

$$= (7.5 \text{ rad/s})\left(\frac{4}{5}\hat{i} - \frac{9}{25}\hat{j} - \frac{12}{25}\hat{k} \text{ in}\right)$$

$$= \underline{6.0\hat{i} - 2.7\hat{j} + 3.6\hat{k} \text{ in/s}}$$

Note: For the following, $\vec{r}$ is the vector <u>from</u> <u>any</u> pt on the axis of rotation <u>to</u> the point you are taking into consideration. (i.e. whose motion is being considered.)

I choose to use $\vec{r}_{E/B}$, that is, the vector r from B to E (E with respect to B).

$$\vec{r}_{E/B} = (0-0)\hat{i} + (0-0)\hat{j} + (0-12)\hat{k} \text{ in}$$

$$= -12\hat{k} \text{ in}$$

$$\vec{v}_E = \vec{\omega}_{AB} \times \vec{r}_{E/B}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6.0 & -2.7 & 3.6 \\ 0 & 0 & -12 \end{vmatrix}$$

$$= 32.4\hat{i} + 72\hat{j} + 0\hat{k} \text{ in/s}$$

$$\underline{\underline{= (2.7\hat{i} + 6\hat{j}) \text{ ft/s}}}$$

Set # 14

2. Continued

Recall: $\vec{\omega}$ is constant. Therefore, $\vec{\alpha} = 0$.

$$\vec{a}_E = (\cancel{\vec{\alpha}_{AB}}^{\,0} \times \vec{r}_{E/B}) + (\vec{\omega}_{AB} \times \vec{V}_E)$$

$$= \vec{\omega}_{AB} \times \vec{V}_E$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6.0 & -2.7 & 3.6 \\ 32.4 & 72 & 0 \end{vmatrix}$$

$$= -259.2\,\hat{i} + 116.64\,\hat{j} + 519.48\,\hat{k}$$

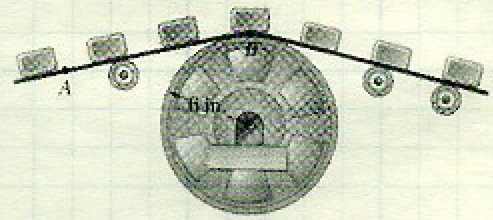
$$= \underline{\underline{-21.6\,\hat{i} + 9.72\,\hat{j} + 43.29\,\hat{k}}}$$

Set #14 – Motion: Translation & Rotation

3. A series of small machine components being moved by a conveyor belt pass over a 6-inch-radius idler pulley. At the instant shown, the velocity of point A is 15 in./s to the left and its acceleration is 9 in./s^2 to the right. Determine

a) the angular velocity and the angular acceleration of the idler pulley,

b) the total acceleration of the machine component at B.



Given: $r = 6 in$

$V_A = 15\ in/s \leftarrow$

$a_A = 9\ in/s^2 \rightarrow$

Find: (a) ω, α of pulley

(b) $\vec{a}_B$

Solution:

With the given information, we can determine that

$\vec{V}_A = \vec{V}_B$

$\vec{a}_{Bt} = \vec{a}_A$

(a) $V_B = \omega r \rightarrow \omega = \frac{V_B}{r}$

$= \frac{15\ in/s}{6 in}$

$= \underline{\underline{2.5\ rad/s}}$

$a_{Bt} = \alpha r \rightarrow \alpha = \frac{a_{Bt}}{r}$

$= \frac{9\ in/s^2}{6\ in}$

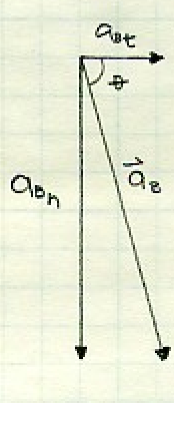
$= \underline{\underline{1.5\ rad/s^2}}$

(b.) $a_{Bt} = 9\ in/s^2 \rightarrow$

$a_{Bn} = \omega^2 r$

$= (2.5\ rad/s)^2 (6\ in)$

$= 37.5\ in/s^2 \downarrow$



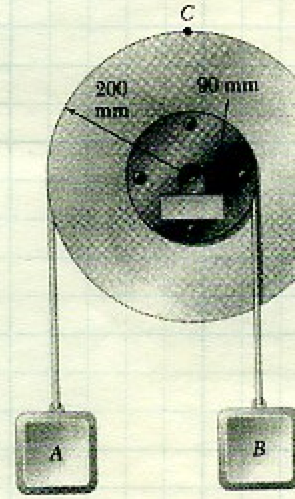
$\vec{a}_B = 38.56\ in/s^2$

$\theta = 76.5°$

Set #14 – Motion: Translation & Rotation

4. Two blocks and a pulley are connected by inextensible cords as shown. Block A has a constant acceleration of 75 mm/s^2 and an initial velocity of 120 mm/s, both directed downward. Determine

a) the number of revolutions executed by the pulley in 6s,
b) the velocity and position of block B after 6s,
c) the acceleration of point C on the rim of the pulley at t = 0.



Given: $a_A = 75 \text{ mm/s}^2 \downarrow$
$v_{A_0} = 120 \text{ mm/s} \downarrow$

Find: (a) revolution in 6 seconds
(b) v_B at 6 seconds
y_B at 6 seconds
(c) $\overline{a}_c$ at t=0

Solution:

(a) $v_A = \omega_o r \longrightarrow \omega_o = \dfrac{v_A}{r}$

$= \dfrac{120 \text{ mm/s}}{200 \text{ mm}}$

$= .6 \text{ rad/s}$ ↻ —①

$a_T = \alpha r \longrightarrow \alpha = \dfrac{a_T}{r}$

$= \dfrac{75 \text{ mm/s}^2}{200 \text{ mm}}$

$= .375 \text{ rad/s}^2$ ↻ —②

Uniformly Accelerated Rotation

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$ (Plug in ①, ②)

$\theta = 0 + (.6 \text{ rad/s})(6 \text{ sec}) + (.5)(.375 \text{ rad/s}^2)(6 \text{ sec})^2$

$= 10.35 \text{ rad}$

$$\left(10.35 \text{ rad}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \underline{\underline{1.647 \text{ rev}}}$$

Set #14

4. continued

(b) UAR: $\omega = \omega_0 + \alpha t$

$= (.6 \text{ rad/s}) + (.375 \text{ rad/s}^2)(6 \text{ sec})$

$= 2.85 \text{ rad/s}$

$V_B = \omega r$

$V_B = (2.85 \text{ rad/s})(90 \text{ mm})$

$V_B = 256.5 \text{ mm/s}$

$S = \theta r$

$= (10.35 \text{ rad})(90 \text{ mm})$

$= 931.5$ mm below initial position

(c) $a_T = \alpha r$

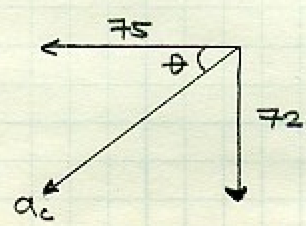
$= (.375 \text{ rad/s}^2)(200 \text{ mm})$

$= 75 \text{ mm/s}^2$

$a_N = \omega^2 r$

$= (.6 \text{ rad/s})^2 (200 \text{ mm})$

$= 72 \text{ mm/s}$



$a_c = 104 \text{ mm/s}^2$

$\theta = 43.8°$