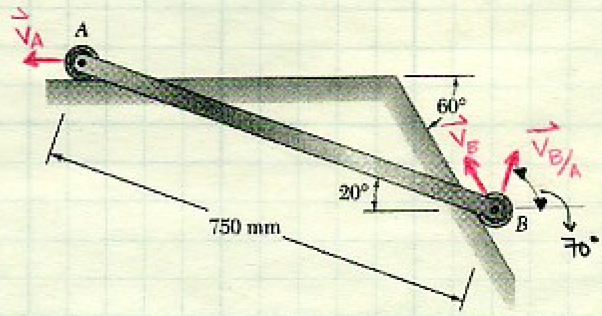
Set #15 – General Plane Motion

1. Small wheels have been attached to the ends of rod AB and roll freely along the surfaces shown. Knowing that wheel A moves to the left with a constant velocity of 1.5 m/s, determine

a) the angular velocity of the rod,
b) the velocity of end B of the rod.

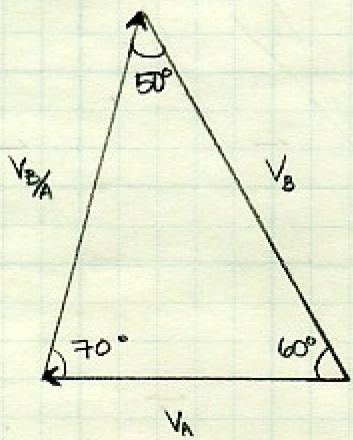


Given: $V_A = 1.5 \text{ m/s} \leftarrow$

Find: (a) ω_{AB}
(b) $\vec{V}_B$

Solution:

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$



$$\frac{V_A}{\sin 50°} = \frac{V_B}{\sin 70°} = \frac{V_{B/A}}{\sin 60°}$$

$$V_B = 1.5 \frac{\sin 70°}{\sin 50°}$$

$$\underline{\underline{V_B = 1.84 \text{ m/s} \ \ 60°}}$$

$$V_{B/A} = 1.5 \frac{\sin 60°}{\sin 50°}$$

$$= 1.696 \text{ m/s} \ \ 70°$$

$$V_{B/A} = \omega r \longrightarrow \omega = \frac{V_{B/A}}{r}$$

$$\omega = \frac{1.696 \text{ m/s}}{.75 \text{ m}}$$

$$\underline{\underline{\omega = 2.26 \text{ rad/s}}}$$

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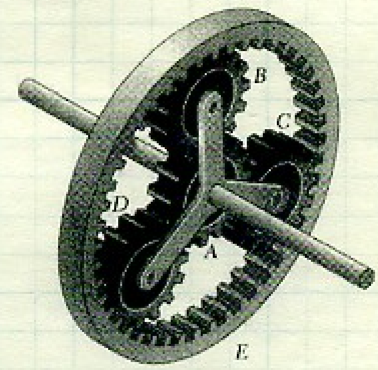
2. In the planetary gear system shown, the radius of gears A, B, C, and D is 3 inches and the radius of the outer gear E is 9 inches. Knowing that gear E has an angular velocity of 120 rpm clockwise and that the central gear has an angular velocity of 150 rpm clockwise, determine

a) the angular velocity of each planetary gear,
b) the angular velocity of the spider connecting the planetary gears.



Given: $r_A, r_B, r_C, r_D = 3 \text{ in}$

$r_E = 9 \text{ in}$

$\omega_E = 120 \text{ rpm} = 12.566 \text{ rad/s}$

$\omega_A = 150 \text{ rpm} = 15.708 \text{ rad/s}$

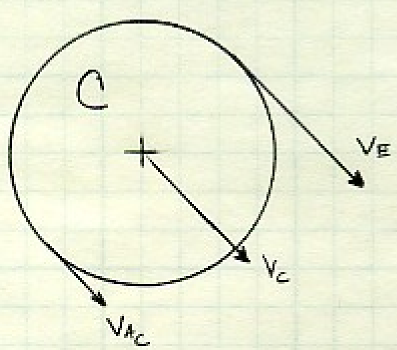
Find: (a.) $\omega_B, \omega_C, \omega_D$

(b.) ω_{spider}

Solution:

$$V_A = V_{A_B} = V_{A_C} = V_{A_D} = \omega_A r = (15.708 \text{ rad/s})(3 \text{ in}) = 47.124 \text{ in/s}$$

$$V_E = V_{E_B} = V_{E_C} = V_{E_D} = \omega_E r = (12.566 \text{ rad/s})(9 \text{ in}) = 113.09 \text{ in/s}$$



$$V_E = V_{A_C} + V_{E/A}$$

$$\rightarrow V_{E/A} = V_E - V_A = 113.09 \text{ in/s} - 47.124 \text{ in/s} = 65.966 \text{ in/s} = \omega r_{E/A}$$

$$\omega = \omega_B = \omega_C = \omega_D = \frac{V_{E/A}}{r_{E/A}}$$

$$\omega = \frac{65.966 \text{ in/s}}{6 \text{ in}}$$

$$\omega = 10.99 \text{ rad/s}$$

$$\underline{\underline{\omega = 105 \text{ rpm}}}$$

Set #15

2. continued

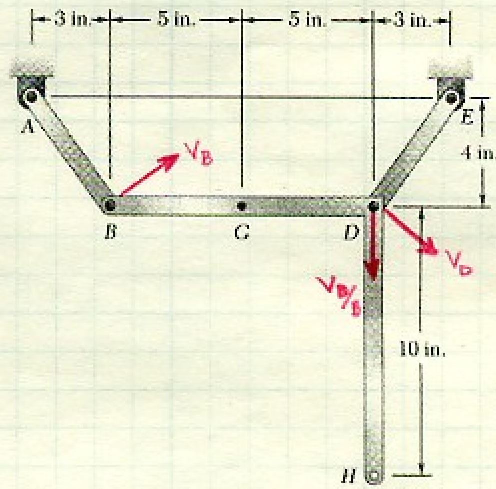
$$V_c = V_{A_c} + V_{c/A_c}$$

$$\longrightarrow \quad V_{c/A} = \omega r_{c/A}$$
$$= (10.99 \text{ rad/s})(3 \text{ in})$$
$$= 32.97 \text{ in/s}$$

$$V_c = V_A + V_{c/A}$$
$$= 47.124 \text{ in/s} + 32.97 \text{ in/s}$$
$$= 80.094 \text{ in/s}$$

$$\omega_c = \frac{V_c}{r}$$
$$\omega_c = \frac{80.094 \text{ in/s}}{6 \text{ in}}$$
$$\omega_c = 13.35 \text{ rad/s}$$
$$\underline{\underline{\omega_c = 127.5 \text{ rpm}}}$$

Set #15 – General Plane Motion



3. In the position shown, bar AB has a constant angular velocity of 20 rad/s counterclockwise. Determine

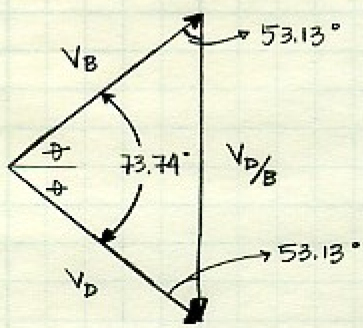
a) the angular velocity of member BDH,
b) the velocity of point G.

Given: $\omega = 20 \text{ rad/s}$ (constant) ↺

Find: (a) ω_{BDH}
(b) V_G

Solution:

$$V_B = \omega_{AB}\, r_{B/A} = (20 \text{ rad/s})(5 \text{ in}) = 100 \text{ in/s}$$



$$V_D = V_B + V_{D/B}$$

$$V_B = V_D = 100 \text{ in/s}$$

$$V_{D/B} = 2(100 \cos 53.13°) = 120 \text{ in/s}$$

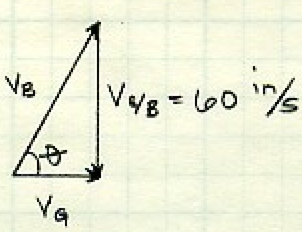
$$V_{D/B} = \omega_{BDH}\, r_{D/B}$$

$$\rightarrow \omega_{BDH} = \frac{V_{D/B}}{r_{D/B}}$$

$$\omega_{BDH} = \frac{120 \text{ in/s}}{10 \text{ in}}$$

$$\underline{\underline{\omega_{BDH} = 12 \text{ rad/s} \circlearrowright}}$$

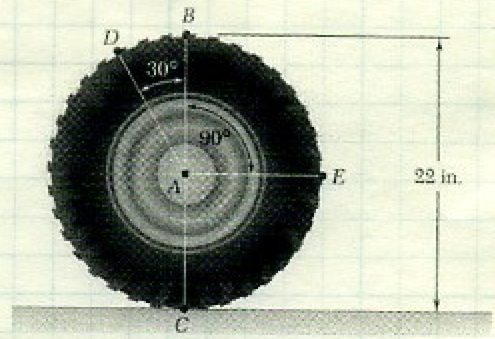
$$\vec{V}_G = \vec{V}_B + \vec{V}_{G/B} = \vec{V}_B + \omega_{BDH}(5 \text{ in})\downarrow = \vec{V}_B + 60 \text{ in/s} \downarrow$$



$$\underline{\underline{V_G = 80 \text{ in/s} \rightarrow}}$$

Set #15 – General Plane Motion

4. An automobile travels to the right at a constant speed of 48 mph. If the diameter of a wheel is 22 inches, determine the velocities of points B, C, D, and E on the rim of the wheel.



Given: $V_A = 48\ mph \rightarrow$
$= 70.4\ ft/s$
$d = 22\ in$
$= 1.833\ ft$
$r = 11\ in$
$= .9167\ ft$

Find: V_B, V_C, V_D, V_E

We know that if there is no slip between the tire and the ground V_C must be 0 $\underline{\underline{(V_C = 0)}}$

$V_A = V_C + V_{A/C}$
$V_A = V_{A/C}$

$\rightarrow V_A = V_{A/C} = \omega r_{A/C}$
$= 76.84\ rad/s$

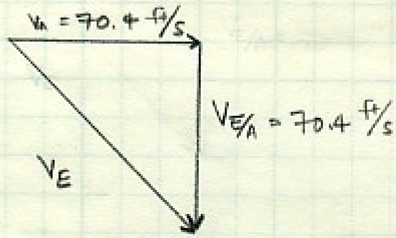
$V_A \quad V_{B/A}$

V_B

$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

$V_{B/A} = \omega r$
$= 70.4\ ft/s \rightarrow$

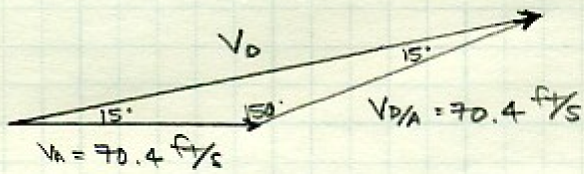
$\vec{V_B} = 70.4\ ft/s \rightarrow + 70.4\ ft/s \rightarrow$
$\underline{\underline{\vec{V_B} = 140.8\ ft/s \rightarrow}}$



$\underline{\underline{\vec{V_E} = 99.56\ ft/s \searrow 45°}}$

Set #15

4. continued



$$\vec{V}_D = \vec{V}_A + \vec{V}_{D/A}$$

$$\vec{V}_D = 2(70.4 \text{ ft/s}) \cos 15°$$

$$\vec{V}_D = 136 \text{ ft/s} \; \angle \; 15°$$