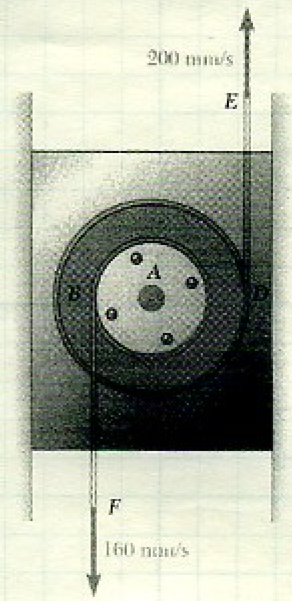
Set #16 – Instantaneous Center

1. A double pulley is attached to a slider block by a pin at A. The 30-mm-radius inner pulley is rigidly attached to the 60-mm-radius outer pulley. Knowing that each of the two cords is pulled at a constant speed as shown, determine

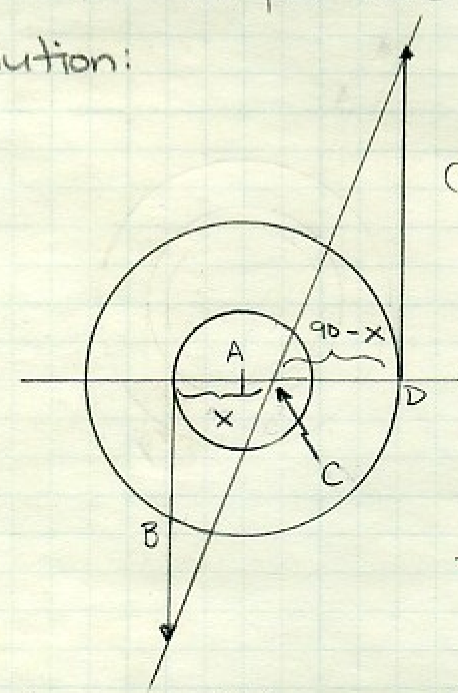
a) the instantaneous center of rotation of the double pulley,
b) the velocity of the slider block,
c) the number of millimeters of cord wrapped or unwrapped on each pulley per second.



Given: $r_i = 30$ mm
$r_o = 60$ mm

Find: (a) instantaneous center of pulley
(b) V_A
(c) mm per second for D and B

Solution:



(a.) Using similar triangles to solve for x.

$$\frac{160}{x} = \frac{200}{90-x}$$

$$1440 - 16x = 20x$$
$$1440 = 36x$$
$$x = 40 \text{ mm}$$

$\Rightarrow$ C is 10mm to the right of A

(b) $\omega = \frac{V_B}{r}$
$= \frac{160 \text{ mm/s}}{40 \text{ mm}}$
$= 4.0$ rad/s

$V_A = \omega r$
$V_A = (4.0 \text{ rad/s})(10 \text{ mm})$
$V_A = 40.0$ mm/s ↓

Set #16

1. continued

(c) $V_{B/A}$ = cord unwrapped at B

$V_B = V_A + V_{B/A}$

$\longrightarrow V_{B/A} = V_B - V_A$

$V_{B/A} = 160\ mm/s \downarrow - 40\ mm/s \downarrow$

$V_{B/A} = -160\ mm/s + 40\ mm/s$

$V_{B/A} = -120\ mm/s$

$\underline{\underline{V_{B/A} = 120\ mm/s \downarrow}}$

$V_{D/A}$ = cord unwrapped at D

$V_{D/A} = V_D - V_A$

$V_{D/A} = 200\ mm/s \uparrow - 40\ mm/s \downarrow$

$V_{D/A} = 200\ mm/s + 40\ mm/s$

$\underline{\underline{V_{D/A} = 240\ mm/s \uparrow}}$

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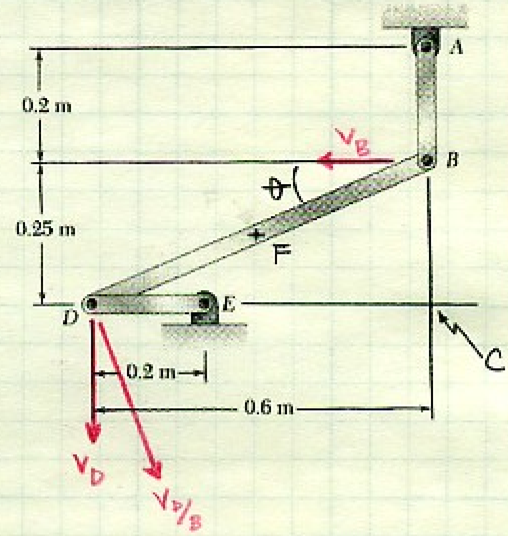
2. Knowing that at the instant shown the angular velocity of rod AB is 15 rad/s clockwise, determine

a) the angular velocity of rod BD,

b) the velocity of the midpoint of rod BD.



Given: $\omega_{AB} = 15\ \text{rad/s}$ (clockwise)

Find: (a) ω_{BD}

(b) V_F

Solution:

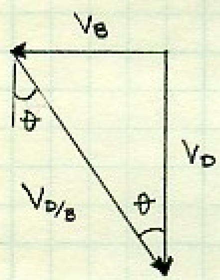
$$\tan\theta = \frac{0.25}{0.6}$$

$$\theta = 22.62^\circ$$

(a) $V_B = \omega_{AB}\, r$

$= (15\ \text{rad/s})(.2\text{m})$

$= 3\ \text{m/s}$



$$\vec{V}_D = \vec{V}_B + \vec{V}_{D/B}$$

$$\frac{V_B}{V_D} = \tan\theta$$

$$\rightarrow V_D = \frac{V_B}{\tan\theta} = 7.2\ \text{m/s}$$

$$V_{D/B} = \frac{V_B}{\sin\theta} = 7.8\ \text{m/s}$$

$$\omega_{DB} = \frac{V_{D/B}}{r}$$

$$\omega_{DB} = \frac{7.8\ \text{m/s}}{0.65\text{m}}$$

$$\omega_{DB} = 12\ \text{rad/s}\ \text{(clockwise)}$$

Set #16

2. continued

(b) $\vec{v}_{F/B} = \omega r$

$= (12 \text{ rad/s})\left(\frac{0.65 \text{ m}}{2}\right)$

$= 3.9 \text{ m/s}$

$\vec{v}_F = \vec{v}_B + \vec{v}_{F/B}$

$\vec{v}_F = 3 \text{ m/s} \leftarrow + 3.9 \text{ m/s}$ ↘ $\theta = 22.62°$

$\vec{v}_F = 3.9 \text{ m/s}$ ↘ $67.4°$

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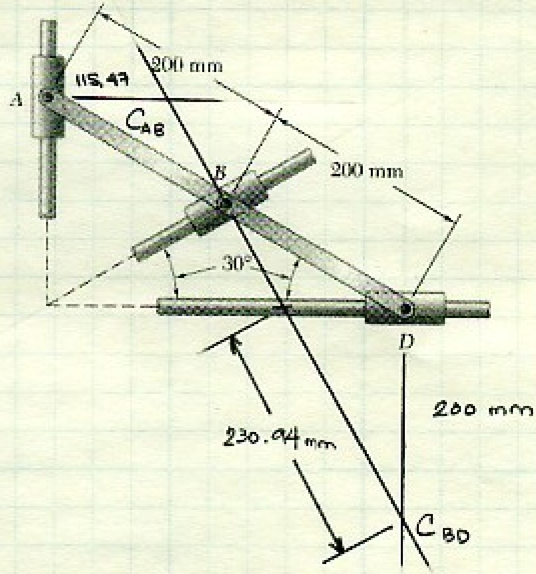
3. Two rods AB and BD are connected to three collars as shown. Knowing that collar A moves downward with a velocity of 120 mm/s determine at the instant shown

a) the angular velocity of each rod,

b) the velocity of collar D.



Given: $V_A = 120\ \text{mm/s}\ \downarrow$

Find: (a) ω_{AB}, ω_{BD}

(b) V_D

Solution:

$$V_A = \omega_{AB}\, r_{A/C}$$

$$\rightarrow \omega_{AB} = \frac{V_A}{r_{A/C}}$$

$$\omega_{AB} = \frac{120\ \text{mm/s}}{115.47\ \text{mm}}$$

$$\underline{\omega_{AB} = 1.03923\ \text{rad/s}\ \circlearrowleft}$$

$$V_B = \omega_{AB}\, r_{B/C}$$
$$= (1.03923\ \text{rad/s})(115.47\ \text{mm})$$
$$= 120\ \text{mm/s}\ \measuredangle\ 30°$$

$$\vec{V}_B = \vec{V}_{B/C_{BD}} = \omega_{BD}\, r_{B/C}$$

$$\rightarrow \omega_{BD} = \frac{120\ \text{mm/s}}{230.94\ \text{mm} + 115.47\ \text{mm}}$$

$$\omega_{BD} = \frac{120\ \text{mm/s}}{346\ \text{mm}}$$

$$\underline{\omega_{BD} = .346\ \text{rad/s}\ \circlearrowright}$$

$$\vec{V}_D = \vec{V}_C + \vec{V}_{D/C}$$
$$\vec{V}_D = 0 + \omega_{BD}\, r_{D/C}$$
$$\vec{V}_D = (.346\ \text{rad/s})(200\ \text{mm})$$
$$\underline{\vec{V}_D = 69.28\ \text{mm/s}\ \rightarrow}$$

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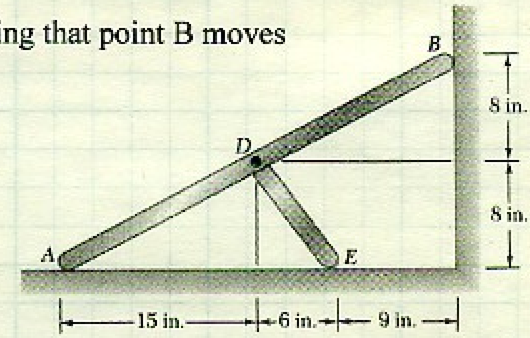
4. Two rods AB and DE are connected as shown. Knowing that point B moves downward with a velocity of 60 inches/s, determine

a) the angular velocity of each rod,

b) the velocity of point E.

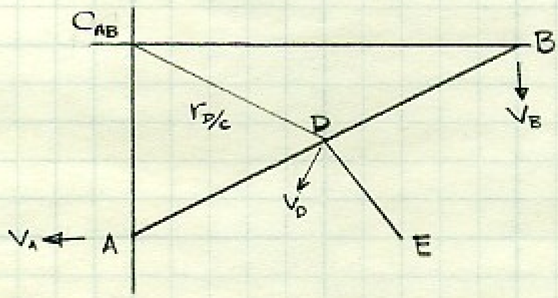


Given: $V_B = 60$ in/s ↓

Find: (a.) ω_{AB}, ω_{DE}

(b.) $\vec{V}_E$

Solution:



$V_B = \omega_{AB} r_{B/c}$

$\longrightarrow \omega_{AB} = \dfrac{60 \text{ in/s}}{30 \text{ in}}$

$\underline{\underline{\omega_{AB} = 2 \text{ rad/s} \circlearrowright}}$

$V_D = \omega_{AB} r_{D/c}$

$= (2 \text{ rad/s})(17 \text{ in})$

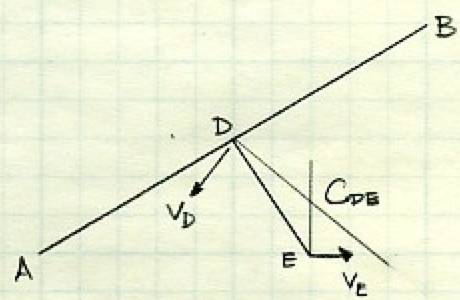
$= 34 \text{ in/s}$ ↙ $61.9°$

$V_D = \omega_{DE} r_{D/c}$

$\longrightarrow \omega_{DE} = \dfrac{V_D}{r_{D/c}}$

$\omega_{DE} = \dfrac{34 \text{ in/s}}{6.8 \text{ in}}$

$\underline{\underline{\omega_{DE} = 5 \text{ rad/s} \circlearrowleft}}$



$V_E = \omega_{DE} r_{E/c}$

$\underline{\underline{\vec{V}_E = 24 \text{ in/s} \longrightarrow}}$