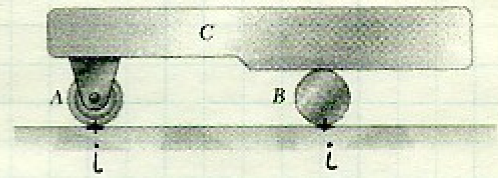
Set #17 – Acceleration in Plane Motion

1. A carriage C is supported by a caster A ad a cylinder B, each of 50-mm diameter. Knowing that at the instant shown the carriage has an acceleration of 2.4 m/s^2 and a velocity of 1.5 m/s, both directed to the left, determine

   a) the angular accelerations of the caster and of the cylinder,

   b) the accelerations of the centers of the caster and of the cylinder.



Given: $d_A = d_B = 50$ mm
$a_c = 2.4\ m/s^2 \leftarrow$
$V_c = 1.5\ m/s \leftarrow$

Find: (a) α_A, α_B
(b) $\vec{a}_A$; $\vec{a}_B$

Solution:

For pin connections, the points have the same acceleration. (Point A is in rectilinear motion.)

$$\rightarrow \vec{a}_A = 2.4\ m/s^2 \leftarrow$$

$$\Delta x_A = \Delta\theta_A r$$
$$V_A = \omega_A r_{A/i}$$
$$a_A = \alpha_A r_{A/i}$$
$$\rightarrow \alpha_A = \frac{2.4\ m/s^2}{.025 m}$$
$$\alpha_A = 96\ rad/s^2 \circlearrowright$$

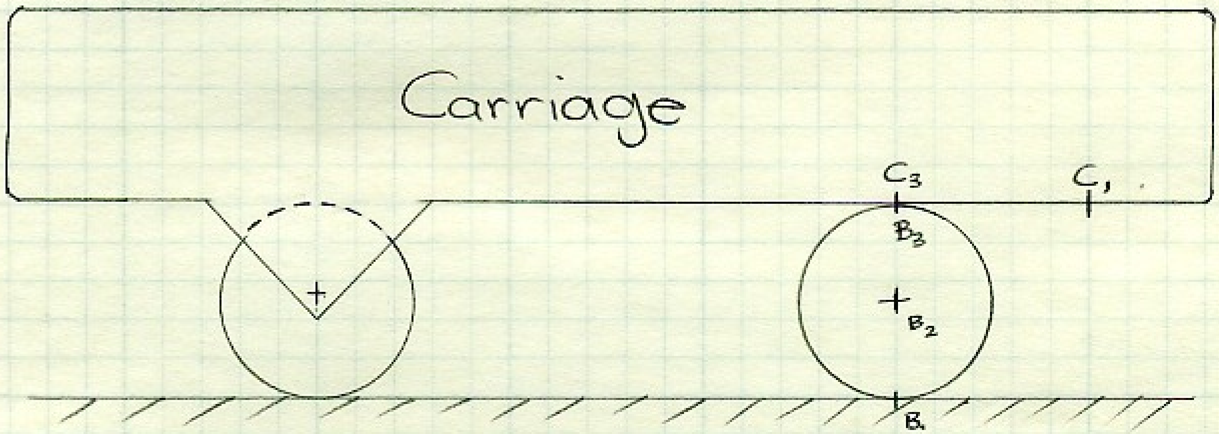
Point B is also in rectilinear motion.

$\Delta x_B = \frac{1}{2} \Delta x_c$ (refer to following page)
$$V_B = \tfrac{1}{2} V_c$$
$$a_B = \tfrac{1}{2} a_c$$
$$\vec{a}_B = 1.2\ m/s^2 \leftarrow$$

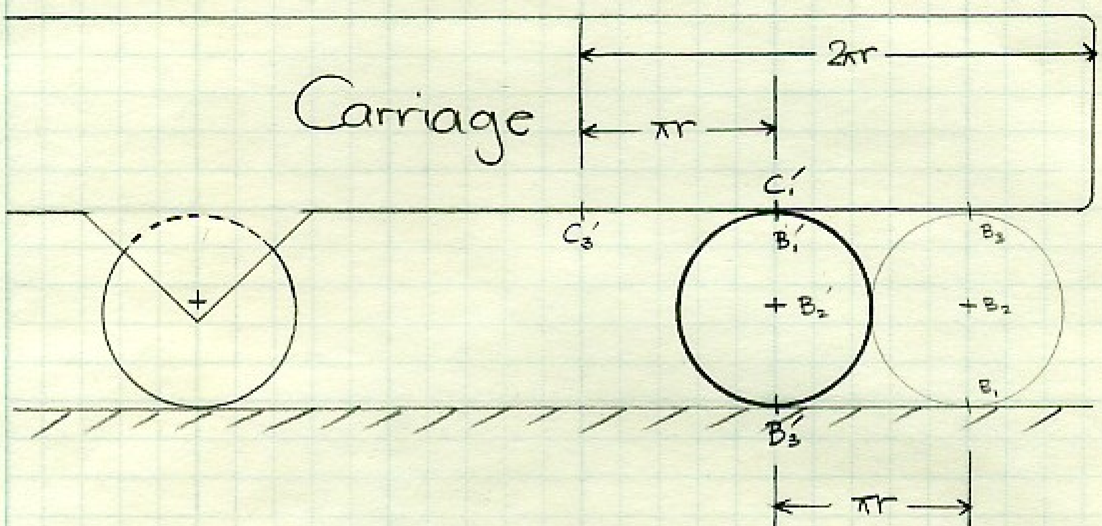
$$\Delta x_B = \Delta\theta\, r_{B/i}$$
$$V_B = \omega_B r_{B/i}$$
$$a_B = \alpha_B r_{B/i}$$
$$\rightarrow \alpha_B = \frac{1.2\ m/s^2}{.025 m}$$
$$\alpha_B = 4.8\ rad/s^2 \circlearrowright$$

Set #17

1. continued



After B rotates through π



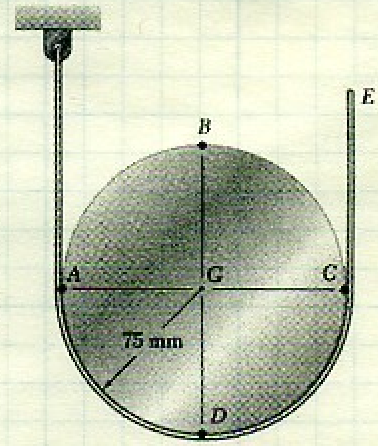
$$\Delta x_c = 2 \Delta x_B$$
$$v_c = 2 v_B$$

Set #17 – Acceleration in Plane Motion

2. The motion of the 75-mm-radius cylinder is controlled by the cord shown. Knowing that end E of the cord has a velocity of 300 mm/s and an acceleration of 480 mm/s^2, both directed upward, determine the acceleration

a) of point A,

b) of point B.



Given: $r = 75$ mm

$\vec{V}_E = 300$ mm/s $\uparrow$

$\vec{a}_E = 480$ mm/s^2 $\uparrow$

Find: (a) $\vec{a}_A$

(b) $\vec{a}_B$

Solution:

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$
$$= \vec{a}_G + [(\vec{\alpha} \times \vec{r}_{A/G}) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/G})] \quad \text{—①}$$

$$\vec{a}_B = \vec{a}_G + [(\vec{\alpha} \times \vec{r}_{B/G}) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/G})] \quad \text{—②}$$

To solve for $\vec{a}_A$ and $\vec{a}_B$, we need $\vec{a}_G$, α, ω

G is in Rectilinear motion

$\vec{a}_G$: $\Delta X_G = \frac{1}{2} \Delta X_E$ (refer to following page)

$V_G = \frac{1}{2} V_E$

$a_G = \frac{1}{2} a_E$

$= 240$ mm/s^2 $\uparrow$

α: $\Delta X_G = \frac{1}{2} \Delta X_E$

$\Delta\theta_G r = \frac{1}{2} \Delta X_E$

$\omega r = \frac{1}{2} V_E$

$\alpha r = \frac{1}{2} a_E$

$\alpha = \dfrac{\frac{1}{2} a_E}{r}$

$= 3.2$ rad/s^2 ↻

Set #17

2. continued

ω: we have 2 points on a rigid body.

$\vec{v}_A = \vec{0}$
$\vec{v}_C = 300 \text{ mm/s} \uparrow$

$\vec{v}_C = \vec{v}_A + \vec{v}_{C/A}$
$\vec{v}_C = 0 + \omega r_{C/A}$
$\rightarrow \omega = \frac{v_C}{r_{C/A}}$
$= \frac{300 \text{ mm/s} \uparrow}{2(75 \text{ mm})}$
$= 2 \text{ rad/s} \circlearrowright$

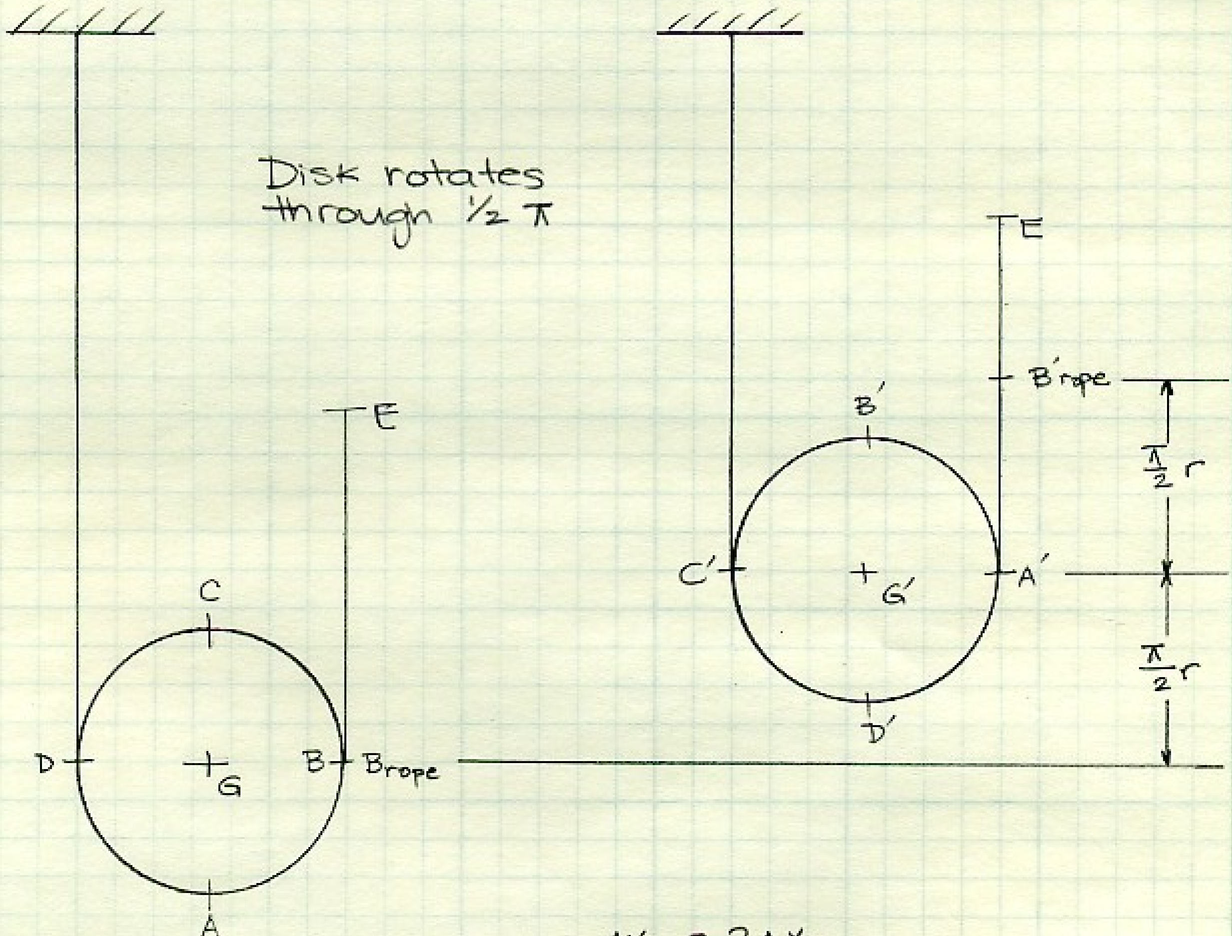
We can now use ① and ②

$\vec{a}_A = 240 \uparrow + (3.2)(75) \downarrow + 2^2(75) \rightarrow$
$\vec{a}_A = 240 \uparrow + 240 \downarrow + 300 \rightarrow$
$\underline{\underline{\vec{a}_A = 300 \text{ mm/s}^2 \rightarrow}}$

$\vec{a}_B = 240 \uparrow + (3.2)(75) \leftarrow + 2^2 (75) \downarrow$
$\vec{a}_B = 60 \downarrow + 240 \leftarrow$
$\underline{\underline{\vec{a}_B = 247 \text{ mm/s}^2 \swarrow 14^\circ}}$

Set #17

2. continued



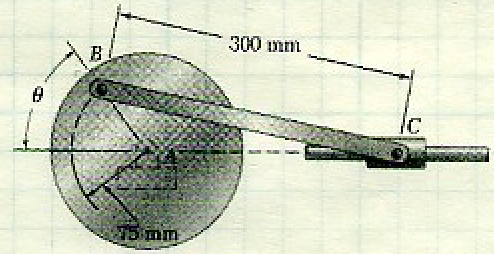
$$\Delta x_E = 2\Delta x_G$$
$$v_E = 2 v_G$$
$$a_E = 2 a_G$$

Alternatively, $x_G + \frac{1}{2}(2\pi r) + (x_G - x_E) =$

$$\Delta x_G + \Delta x_G - \Delta x_E = 0$$
$$2\Delta x_G = \Delta x_E$$
$$2v_G = v_E$$
$$2a_G = a_E$$

Set #17 – Acceleration in Plane Motion

3. The disk shown has a constant angular velocity of 360 rpm clockwise. Determine the acceleration of collar C when $\theta = 90°$.



Given: $\omega_{AB} = 360$ rpm (constant)
$= 37.7$ rad/s
$\alpha_{AB} = 0$

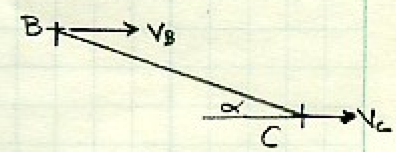
Find: $\vec{a}_c$ when $\theta = 90°$

Solution:

$$\vec{a}_c = \vec{a}_B + \vec{a}_{c/B}$$
$$= \vec{a}_B + [\vec{\alpha}_{BC} \times \vec{r}_{c/B} + \vec{\omega}_{CB} \times (\vec{\omega}_{CB} \times \vec{r}_{c/B})] \quad \text{—①}$$

$\vec{a}_B$:
$$\vec{a}_B = [(\alpha_{AB} \times \vec{r}_{B/A}) + (\vec{\omega}_{AB} \times (\omega_{AB} \times r_{B/A}))]$$
$$\vec{a}_B = \alpha r + \omega^2 r \downarrow$$
$$\vec{a}_B = 0 + \omega^2 r \downarrow$$
$$\vec{a}_B = (37.7 \text{ rad/s})^2 (75 \text{ mm}) \downarrow$$
$$\vec{a}_B = 106.6 \text{ mm/s}^2 \downarrow$$

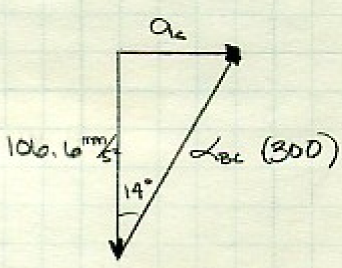
ω_{CB}:
$$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$$
$$\vec{V}_B = (37.7 \text{ rad/s})(75 \text{ mm}) \rightarrow$$
$$\vec{V}_B = 2.83 \text{ mm/s} \rightarrow$$



$$\vec{V}_c = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{c/B}$$

NOTE: This can't work if $\omega_{BC} \neq 0$.
Therefore, $\omega_{BC} = 0$!
→ BC is in translation at this instant.

Using ①,
$$\vec{a}_c = 106.6 \downarrow + \alpha_{BC} r_{c/B} \; 14° + 0$$
$$= 106.6 \downarrow + \alpha_{BC} (300) \; 14°$$



$$\vec{a}_c = 106.6 \tan 14°$$
$$\underline{\underline{a_c = 27.5 \text{ mm/s}^2 \rightarrow}}$$