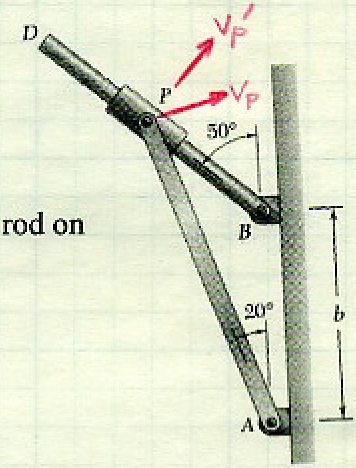
Set #18 – Coriolis Acceleration



1. Two rotating rods are connected by a slider block P. The rod attached at B rotates with a constant clockwise angular velocity ω_B. For $b = 10$ inches and $\omega_B = 5$ rad/s, determine for the position shown

a) the angular velocity of the rod attached at A,

b) the relative velocity of the slider block P with respect to the rod on which it slides.

Given: $\omega_B = 5 \text{ rad/s}$ ↻ (constant)

$b = 10$

Find: (a) ω_A

(b) $\vec{V}_{P/DB} = \vec{V}_{P/\mathcal{F}}$

Solution:

We should first get the geometry of the problem

$$\frac{AP}{\sin 20^\circ} = \frac{10}{\sin 30^\circ} = \frac{PB}{\sin 20^\circ}$$

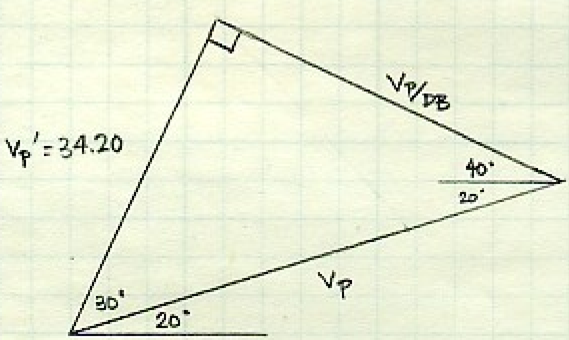
$$AP = 15.32 \text{ in}$$
$$PB = 6.84 \text{ in}$$

Now, consider DB to be the rotating frame ($\mathcal{F}$)

$$\vec{V}_P = \vec{V}_{P'} + \vec{V}_{P/DB}$$

$$V_P \measuredangle 20^\circ = V_P' \measuredangle 50^\circ + V_{P/DB} \measuredangle 40^\circ$$

$$V_P' = (\omega_{DB})(PB)$$
$$= (5 \text{ rad/s})(6.84 \text{ in})$$
$$= 34.20 \text{ in/s}$$



$$V_{P/DB} = V_P' \tan 30^\circ$$
$$\underline{\underline{V_{P/DB} = 19.75 \text{ m/s}}}$$

$$V_P' = V_P \cos 30^\circ$$
$$V_P = \frac{V_P'}{\cos 30^\circ}$$
$$= 39.49 \text{ in/s}$$

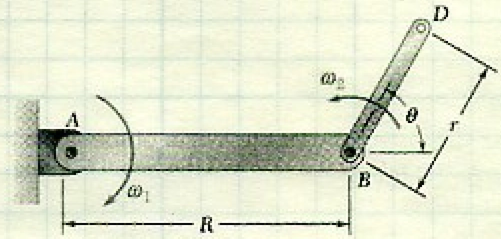
$$V_P = \omega_A r_{P/A} \longrightarrow \omega_A = \frac{V_P}{r_{P/A}}$$

$$\omega_A = \frac{39.49}{15.32}$$

$$\underline{\underline{\omega_A = 2.58 \text{ rad/s}}}$$

Set #18 – Coriolis Acceleration

2. Rod AB of length R = 15 inches rotates about A with a constant clockwise angular velocity ω_1 of 5 rad/s. At the same time, rod BD of length r = 8 inches rotates about B with a constant counterclockwise angular velocity ω_2 of 3 rad/s with respect to rod AB. Knowing that $\theta = 60^\circ$, determine for the position shown the acceleration of point D.



Given: $R = 15$ in
$\omega_1 = 5$ rad/s ↻ (constant)
$r = 8$ in
$\omega_2 = 3$ rad/s ↺ (constant)
$\theta = 60^\circ$

Find: $\vec{a}_D$

Solution:

Let consider AB to be the rotating frame ($\mathcal{F}$)

$$\vec{a}_D = \vec{a}_{D'} + \vec{a}_{D/\mathcal{F}} + \vec{a}_c \quad -(1)$$

First, $\vec{r}_{D/A} = (15 + 8\cos 60^\circ)\,\hat{i} + 8\sin 60^\circ\,\hat{j}$
$= 19\hat{i} + 6.928\hat{j}$
$\vec{r}_{D/B} = 4\hat{i} + 6.928\hat{j}$

$$\vec{a}_{D'} = \cancel{\alpha_{AB}}^{\,0} \times r_{D/A} + \omega_{AB} \times (\omega_{AB} \times r_{D/A})$$
$$= -5\hat{k} \times [(-5\hat{k}) \times (19\hat{i} + 6.928\hat{j})]$$
$$= -\omega^2 \vec{r}$$
$$= -25\,(19\hat{i} + 6.928\hat{j})$$
$$= -475\hat{i} - 172.2\hat{j}$$

$$\vec{a}_c = 2\,\vec{\omega}_1 \times \vec{v}_{D/\mathcal{F}}$$

$$\rightarrow \vec{v}_{D/\mathcal{F}} = \vec{\omega}_{BD} \times \vec{r}_{D/B}$$
$$= 3\hat{k} \times (4\hat{i} + 6.928\hat{j})$$
$$= -20.78\hat{i} + 12\hat{j}$$

$$\vec{a}_c = 2(-5\hat{k}) \times (-2.78\hat{i} + 12\hat{j})$$
$$= 120\hat{i} + 207.8\hat{j}$$

Set #18

2. continued

Because we no longer have unknowns, we can now plug all the values obtained into equation ①.

$$\vec{a}_D = \vec{a}_D' + \vec{a}_{D/\mathcal{F}} + \vec{a}_c$$

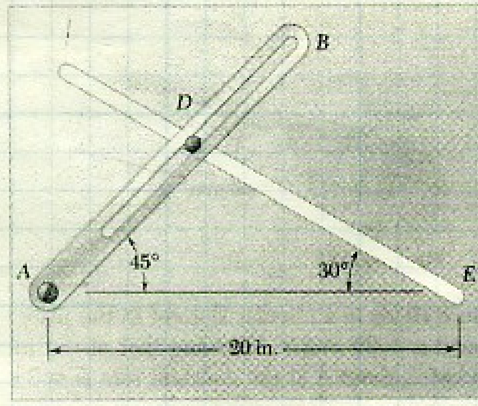
$$\vec{a}_D = (-475\hat{\imath} - 173.2\hat{\jmath}) + (-36\hat{\imath} - 62.35\hat{\jmath}) + (120\hat{\imath} + 207.8\hat{\jmath})$$

$$\underline{\underline{\vec{a}_D = (391\hat{\imath} - 27.75\hat{\jmath})\ \text{in}/\text{s}^2}}$$

$$\underline{\underline{\vec{a}_D = 391\ \text{in}/\text{s}^2 \ \searrow 4.06^\circ}}$$

Set #18 – Coriolis Acceleration

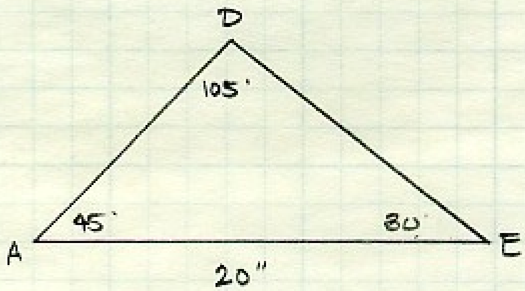
3. The motion of pin D is guided by a slot cut in rod AB and a slot cut in the fixed plate. Knowing that at the instant shown rod AB rotates with an angular velocity of 3 rad/s and an angular acceleration of 5 rad/s^2, both counterclockwise, determine the acceleration of pin D.



Given: $\omega_{AB} = 3\ \mathrm{rad/s}$ ↺
$\alpha_{AB} = 5\ \mathrm{rad/s^2}$ ↺

Find: $\vec{a}_D$

Solution: consider AB to be the rotating frame $\mathcal{F}$



$$\frac{20}{\sin 105^\circ} = \frac{DE}{\sin 45^\circ} = \frac{AD}{\sin 30^\circ}$$

$AD = 10.353''$
$DE = 14.641''$

$$\vec{a}_D = \vec{a}_{D'} + \vec{a}_{D/\mathcal{F}} + \vec{a}_c \quad \text{—}(1)$$

$$\vec{a}_{D'} = \underbrace{\vec{\alpha}_{AB} \times \vec{r}_{D/A}}_{a_{D't}} + \underbrace{\vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{D/A})}_{a_{D'n}}$$

$$= (5)(10.353) + (3^2)(10.353)$$
$$= 51.736\ \mathrm{in/s^2}\ \measuredangle 45^\circ + 93.177\ \measuredangle 45^\circ$$

$$\vec{a}_{D/\mathcal{F}} = a_{D/\mathcal{F}}\ \measuredangle 45^\circ$$

$$\vec{a}_c = 2\,\vec{\omega}_{AB} \times \vec{v}_{D/\mathcal{F}}$$

Need to find $\vec{v}_{D/\mathcal{F}}$

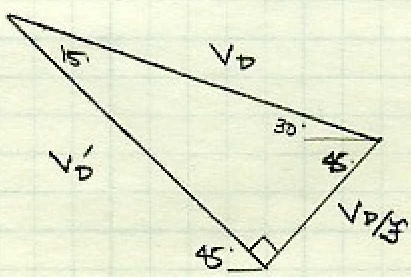
Set #18

3. continued

$$\vec{V}_D = \vec{V}_D' + \vec{V}_{D/F}$$

$$V_D' = \omega_{AB}\, r_{D/A}$$
$$= 3(10.353)$$
$$= 31.059 \angle 45°$$

$$\vec{V}_D \angle 30° = 31.059 \angle 45° + V_{D/F} \angle 45°$$



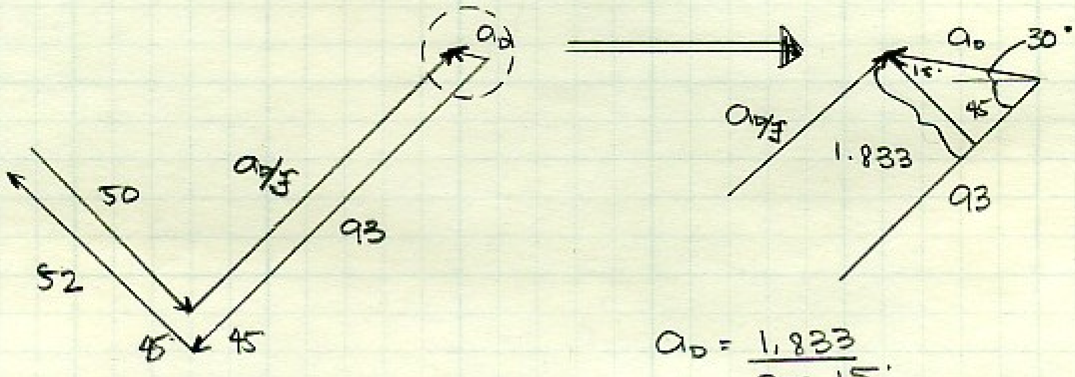
$$V_D = \frac{31.06}{\cos 15°}$$
$$= 32.15 \text{ in/s}$$

$$V_{D/F} = 31.06 \tan 15°$$
$$= 8.322 \text{ in/s}$$

$$\vec{a}_c = 2\, \vec{\omega}_{AB} \times \vec{V}_{D/F}$$
$$= 2(3)(8.322)$$
$$= 49.932 \text{ in/s}^2 \angle 45°$$

Substituting all known values into ①

$$a_D \angle 30° = 51.763 \angle 45° + 93.177 \angle 45°$$
$$+ a_{D/F} \angle 45° + 49.932 \angle 45°$$



$$a_D = \frac{1.833}{\cos 15°}$$

$$\underline{\underline{a_D = 1.898 \text{ in/s}^2 \angle 30°}}$$