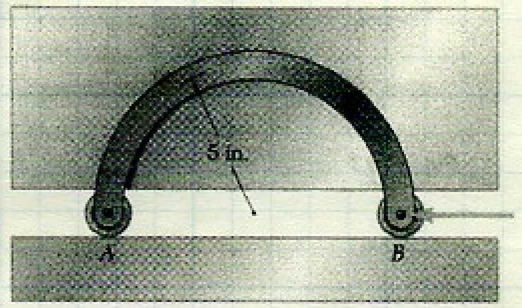
Set #19 – Plane Motion of Rigid Bodies

1. The motion of the 3-lb rod AB is guided by two small wheels that roll freely in a horizontal slot cut in a vertical plate. Determine

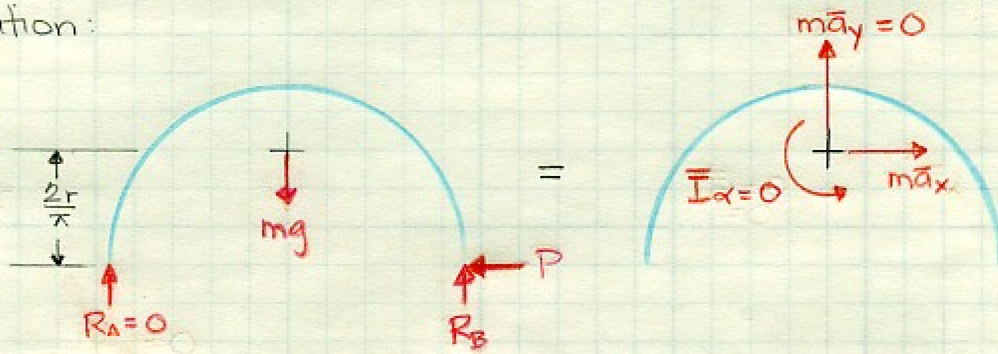
a) the force **P** for which the reaction at A is zero,
b) the corresponding acceleration of the rod.



Given: $W = 3\,\text{lb}$
$r = 5\,\text{in}$

Find: (a) P for $R_A = 0$
(b) $\bar{a}$

Solution:



$$\Sigma F_x = m\bar{a}_x$$
$$-P = ma_x \quad \text{— (1)}$$

$$\Sigma F_y = m\bar{a}_y$$
$$R_A + R_B - W = 0$$
$$0 + R_B - W = 0$$
$$R_B = W \quad \text{— (2)}$$

$$+\circlearrowleft \Sigma M_G = \bar{I}\alpha$$
$$-R_A r + R_B r - P\left(\frac{2r}{\pi}\right) = 0$$
$$0 + R_B r - P\left(\frac{2r}{\pi}\right) = 0$$
$$R_B r - P\left(\frac{2r}{\pi}\right) = 0 \quad \text{— (3)}$$

Set #19

1. continued

② into ③

$$Wr - P\left(\frac{2r}{\pi}\right) = 0$$

$$P = \left(\frac{\pi}{2r}\right) Wr$$

$$= \frac{W\pi}{2}$$

$$= \frac{(3\,\text{lb})\pi}{2}$$

$$\underline{\underline{P = 4.71\ \text{lbs}}}$$

We can now solve for $\bar{a}_x$ in ①

$$-P = m\bar{a}_x \rightarrow \bar{a}_x = \frac{-P}{m}$$

$$= \frac{-4.71\,\text{lb}}{\left(\frac{3\,\text{lb}}{32.2}\right)}$$

$$= -50.58\ \text{ft}/\text{s}^2$$

$$\underline{\underline{\bar{a}_x = 50.58\ \text{ft}/\text{s}^2 \leftarrow}}$$

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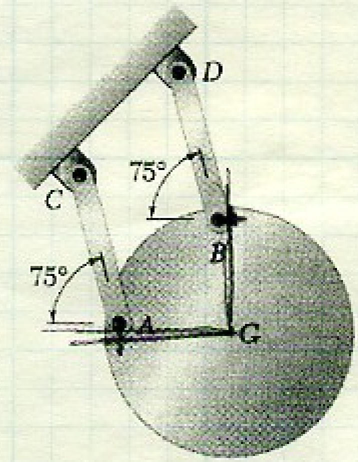


2. A uniform circular plate of mass 3 kg is attached to two links AC and BD of the same length. Knowing that the plate is released from rest in the position shown, determine

a) the acceleration of the plate,

b) the tension in each link.

Given: $m = 3\text{kg}$

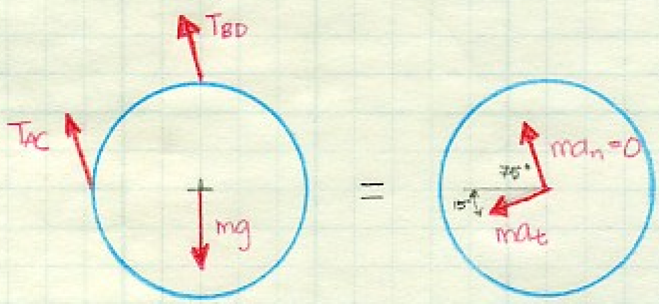
$v_o = 0$

Find: (a) $\vec{a}_G$

(b.) T_{AC} , T_{BD}

Solution: The plate is in translation only.

$\rightarrow$ The path of G is known.



$\vec{a}_G = \vec{a}_{G_t} + \vec{a}_{G_n}$

$\vec{a}_G = \vec{a}_t + 0$

Note: $\vec{a}_{G_n} = 0$ because $\frac{v^2}{r} = 0$

$\Sigma F_t = ma_t$

$\cancel{m}g \cos 75° = \cancel{m}a_t$

$\rightarrow a_t = g \cos 75°$

$\underline{a_t = 2.54 \text{ m/s}^2 \;\measuredangle\; 15°}$

$\Sigma M_B = (ma_t \cos 15°)r$

$(T_{AC} \cos 75°)\cancel{r} + (T_{AC} \sin 75°)\cancel{r} = (ma_t \cos 15°)\cancel{r}$

$$\rightarrow T_{AC} = \frac{ma_t \cos 15°}{\cos 75° + \sin 75°}$$

$\underline{T_{AC} = 6.01 \text{ N}}$

$\Sigma M_A = (ma_t \cos 75°)r$

$mg\cancel{r} - (T_{BC} \cos 75°)\cancel{r} - (T_{BD} \sin 75°)\cancel{r} = (ma_t \cos 75)\cancel{r}$

$$\rightarrow T_{BC} = \frac{ma_t \cos 75 - mg}{\cos 75° + \sin 75°}$$

$\underline{T_{BC} = 22.4 \text{N}}$

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3. It takes 10 min for a 6000-lb flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 36 inches, determine the average magnitude of the couple due to kinetic friction in the bearings.

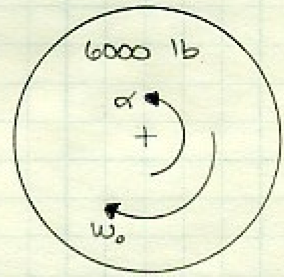
Given: $\omega_o = 300 \text{ rpm} = 31.416 \text{ rad/s}$

$\omega_f = 0$

$t = 10 \text{ min} = 600 \text{ sec}$

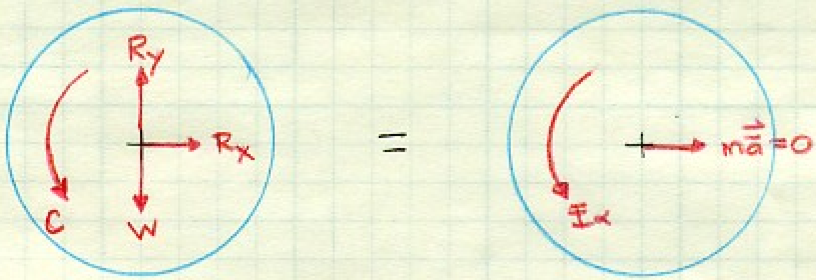
$\bar{k} = 36 \text{ in} = 3 \text{ ft.}$

$W = 6000 \text{ lb}$



Find: $C_{AVE}^{friction}$

Solution:



$$\bar{I} = m\bar{k}^2$$
$$= \left(\frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3 \text{ ft})^2$$
$$\bar{I} = 1677 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\omega_f = \omega_o + \alpha t$$
$$0 = 31.416 \text{ rad/s} + \alpha (600 \text{ sec})$$
$$\longrightarrow \alpha = -0.0524 \text{ rad/s}^2$$

$$\Sigma M_G = \bar{I}\alpha$$
$$C = \bar{I}\alpha$$
$$= (1677 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(-0.0524 \text{ rad/s}^2)$$
$$= -87.8 \text{ ft} \cdot \text{lb}$$

$$\underline{\underline{C_{AVE}^{friction} = 87.8 \text{ ft} \cdot \text{lb}}}$$

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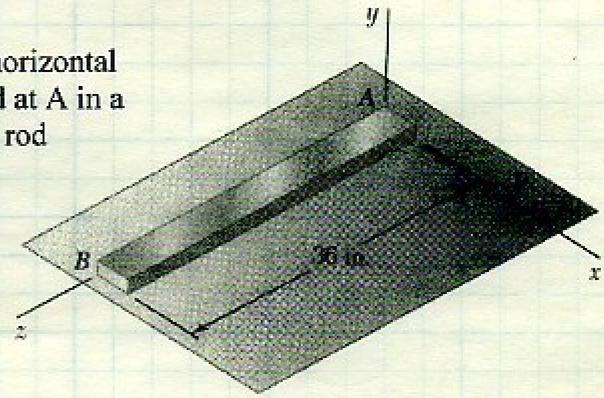
4. A uniform slender rod AB rests on a frictionless horizontal surface, and a force **P** of magnitude 0.25 lb is applied at A in a direction perpendicular to the rod. Knowing that the rod weighs 1.75 lb, determine the acceleration of

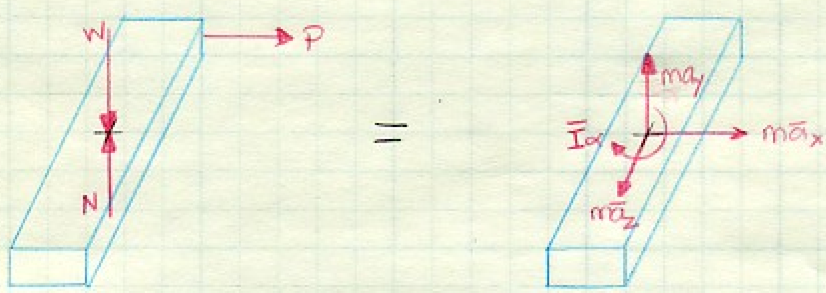
a) point A,
b) point B.



Given: $V_o = 0$
$P = 0.25 \text{ lb}$
$W = 1.75 \text{ lb}$
Frictionless

Find: (a) $\vec{a}_A$
(b) $\vec{a}_B$

Solution:



$$\Sigma F_x = m\bar{a}_x$$
$$P = m\bar{a}_x$$
$$\rightarrow \bar{a}_x = \frac{P}{m} = \frac{0.25 \text{ lb}}{\left(\frac{1.75 \text{ lb}}{32.2 \text{ ft/s}^2}\right)} = 4.6 \text{ ft/s}^2 \longrightarrow$$

$$\Sigma F_y = m\bar{a}_y$$
$$N - W = m\bar{a}_y$$
$$0 = m\bar{a}_y$$
$$\rightarrow \bar{a}_y = 0$$

$$\Sigma F_z = m\bar{a}_z$$
$$0 = m\bar{a}_z$$
$$\rightarrow \bar{a}_z = 0$$

Set # 19

4. continued

$$\bar{I} = \frac{1}{12} m L^2$$
$$= \frac{1}{12} \left(\frac{1.75 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (3 \text{ ft})^2$$
$$= 0.04076 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\Sigma M_G = \bar{I} \alpha$$
$$P(1.5 \text{ ft}) = \bar{I} \alpha$$

$$\rightarrow \alpha = \frac{(0.25 \text{ lb})(1.5 \text{ ft})}{0.04076 \text{ lb} \cdot \text{ft} \cdot \text{s}^2}$$
$$= 9.2 \text{ rad/s}^2$$

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$

$$\vec{a}_G = 4.6 \text{ ft/s}^2 \ \hat{i}$$
$$\vec{a}_{A/G} = (\vec{a}_{A/G})_t + (\vec{a}_{A/G})_n$$

$$(\vec{a}_{A/G})_t = \alpha \, r_{A/G}$$
$$= (9.2)(1.5)$$
$$= 13.8 \ \hat{i}$$

$$\vec{a}_A = 4.6 \text{ ft/s}^2 \ \hat{i} + 13.8 \text{ ft/s}^2 \ \hat{i}$$
$$\underline{\underline{\vec{a}_A = 18.4 \text{ ft/s}^2 \ \hat{i}}}$$

$$\vec{a}_B = \vec{a}_G - \vec{a}_{A/G}$$
$$\vec{a}_B = 4.6 \text{ ft/s}^2 \ \hat{i} - 13.8 \text{ ft/s}^2 \ \hat{i}$$
$$\underline{\underline{\vec{a}_B = -9.2 \text{ ft/s}^2 \ \hat{i}}}$$