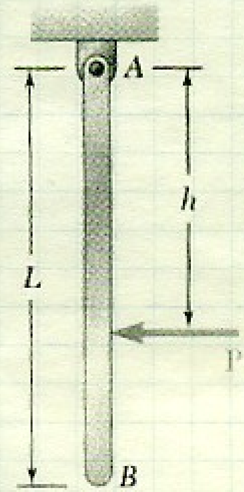
Set #20 – Constrained Plane Motion

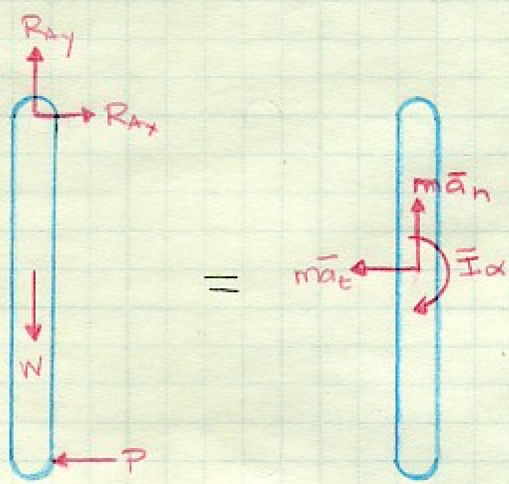
1. A uniform slender rod of length L = 36 inches and weight W = 4 lb hangs freely from a hinge at A. If a force **P** of magnitude 1.5 lb is applied at B horizontally to the left (h = L), determine
   a) the angular acceleration of the rod,
   b) the components of the reaction at A.



Given: $L = 36 \text{ in} = 3\text{ft}$
$W = 4\text{lb}$
$P = 1.5 \text{ lb}$
$h = L = 36 \text{ in} = 3\text{ft}$

Find: (a) α
(b) R_{Ax}, R_{Ay}

Solution:



$\Sigma F_x = m\bar{a}_x$
$R_{Ax} - P = -m\bar{a}_t$

$\rightarrow R_{Ax} = P - m\bar{a}_t$

(NOTE: $a_t = \alpha r$)

$R_{Ax} = P - m\alpha r$ — (1)

$\Sigma F_y = m\bar{a}_y$
$R_{Ay} - W = 0$

$\rightarrow R_{Ay} = W$
$\underline{\underline{R_{Ay} = 4 \text{ lb} \uparrow}}$

Set #20

1. continued

$$\Sigma M_G = \Sigma M_{G\,eff}$$
$$\Sigma M_G = \bar{I}\alpha$$
$$\frac{L}{2} R_{Ax} + \frac{L}{2} P = \bar{I}\alpha \quad \text{— (2)}$$

Substitute (1) into (2).

$$\frac{L}{2}(P - m\alpha r) + \frac{L}{2} P = \left(\frac{1}{12} mL^2\right)\alpha$$
$$\frac{L}{2} P - \frac{L}{2} m\alpha r + \frac{L}{2} P = \frac{1}{12} mL^2 \alpha$$
$$LP - \frac{L}{2} m\alpha r = \frac{1}{12} mL^2 \alpha$$
$$LP - \frac{L}{2} m\alpha \frac{L}{2} = \frac{1}{12} mL^2 \alpha$$
$$LP - \frac{L^2}{4} m\alpha = \frac{1}{12} mL^2 \alpha$$
$$LP - \frac{1}{4} mL^2 \alpha = \frac{1}{12} mL^2 \alpha$$
$$P = \frac{1}{3} mL\alpha$$

$$\longrightarrow \alpha = \frac{3P}{mL} = \frac{3(1.5\text{ lb})}{\left(\frac{4\text{ lb}}{32.2\text{ ft/s}^2}\right)(3\text{ ft})} = 12.075 \text{ rad/s}^2$$

We can now solve for R_{Ax} in equation (1).

$$R_{Ax} = P - m\alpha r$$
$$= 1.5\text{ lb} - \left(\frac{4\text{ lb}}{32.2\text{ ft/s}^2}\right)(12.075\text{ rad/s}^2)(1.5\text{ ft})$$
$$= -0.75\text{ lb}$$

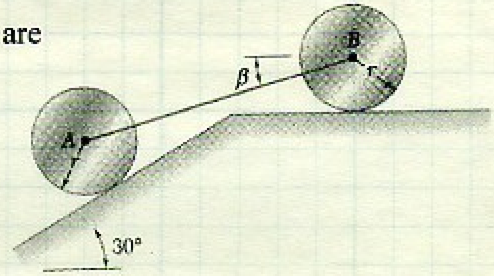
$$\underline{\underline{R_{Ax} = 0.75\text{ lb} \leftarrow}}$$

Set #20 – Constrained Plane Motion

2. Two uniform disks A and B, each of mass m and radius r, are connected by an inextensible cable and roll without sliding on the surfaces shown. Knowing that system is released from rest when $\beta = 15^\circ$, determine the acceleration of the center of

a) disk A,

b) disk B.



Given: $m_A = m_B = m$

$r_A = r_B = r$

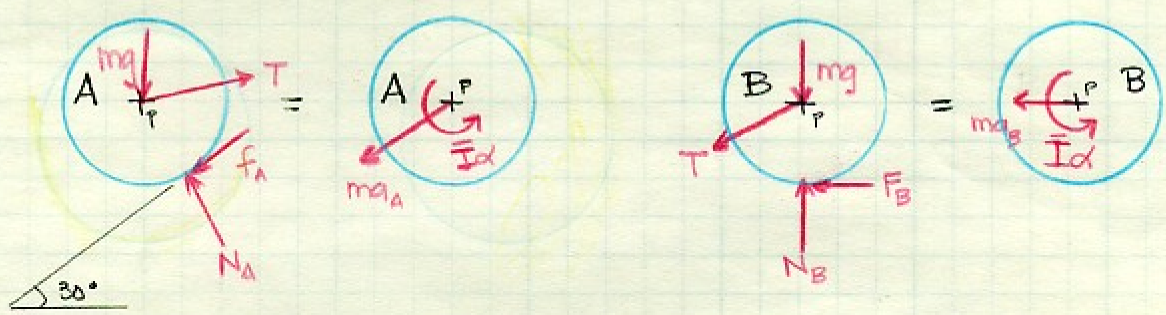
A and B roll with no slip

$v_0 = 0$ at $\beta = 15^\circ$

Find: (a.) $\vec{a}_A$

(b.) $\vec{a}_B$

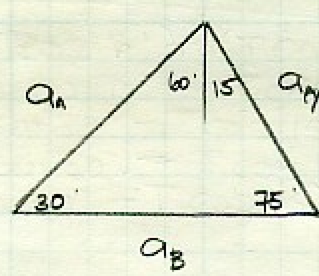
Solution:



$a_A = \alpha_A r$

$a_B = \alpha_B r$

NOTE: Generally, each disk has a different α. However, in this case <u>only</u> ($\beta = 15^\circ$, $\theta = 30^\circ$) $\alpha_A = \alpha_B$



$a_B = a_A$

$r\alpha_B = r\alpha_A$

$\alpha_B = \alpha_A$

isosceles

Set # 20

2. continued

A: $\circlearrowleft^{+} \Sigma M_P = \Sigma M_{P_{eff}}$

$$(mg \sin 30°)r - (T \cos 15°)r = ma_A r + \bar{I}\alpha_A$$
$$mg \sin 30° r - T \cos 15° r = mr^2\alpha_A + \bar{I}\alpha_A$$
$$r(mg \sin 30° - T \cos 15°) = (mr^2 + \bar{I})\alpha_A$$
$$r(mg \sin 30° - T \cos 15°) = (mr^2 + \bar{I})\alpha_A$$
$$r(mg \sin 30° - T \cos 15°) = (mr^2 + \tfrac{1}{2}mr^2)\alpha_A$$
$$mg \sin 30° - T \cos 15° = \tfrac{3}{2} mr\alpha_A \quad (1)$$

B: $\circlearrowleft^{+} \Sigma M_P = \Sigma M_{P_{eff}}$

$$T \cos 15° r = ma_B r + \bar{I}\alpha_B$$
$$T \cos 15° r = m\alpha_B r^2 + \tfrac{1}{2} mr^2\alpha_B$$
$$T \cos 15° r = \tfrac{3}{2} mr^2\alpha_B$$
$$T \cos 15° = \tfrac{3}{2} mr\alpha_B \quad (2)$$

(2) into (1)

$$mg \sin 30° - (\tfrac{3}{2} mr\alpha_B) = \tfrac{3}{2} mr\alpha_A$$
$$g \sin 30° = \tfrac{3}{2} r(\alpha_A + \alpha_B)$$
$$g \sin 30° = \tfrac{3}{2} r(2\alpha)$$
$$g \sin 30° = 3r\alpha$$

$$\longrightarrow \alpha = \frac{g}{6r}$$

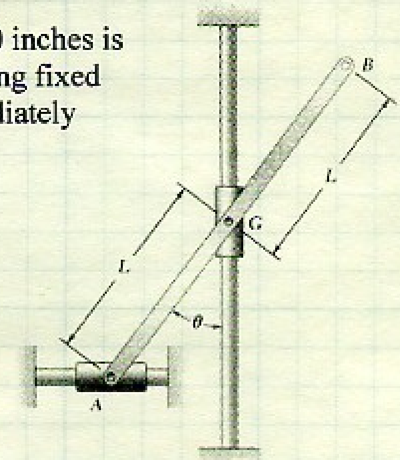
$$\vec{a}_A = \vec{a}_B = r\alpha = r\left(\frac{g}{6r}\right)$$

$$\vec{a}_A = \vec{a}_B = \frac{g}{6}$$

Set #20 – Constrained Plane Motion

3. The uniform rod AB of weight W = 14 lb and total length 2L = 30 inches is attached to collars of negligible weight that slide without friction along fixed rods. If rod AB is released from rest when $\theta = 30^{\circ}$, determine immediately after release

a) the angular acceleration of the rod,
b) the reaction at A.



Given: $W = 14\text{ lb}$
$2L = 30\text{ in} = 2.50\text{ ft}$
$L = 15\text{ in} = 1.25\text{ ft}$
Frictionless
Release from rest when $\theta = 30^{\circ}$

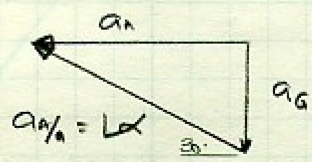
Find: (a) α
(b) R_A

Solution:

(a)

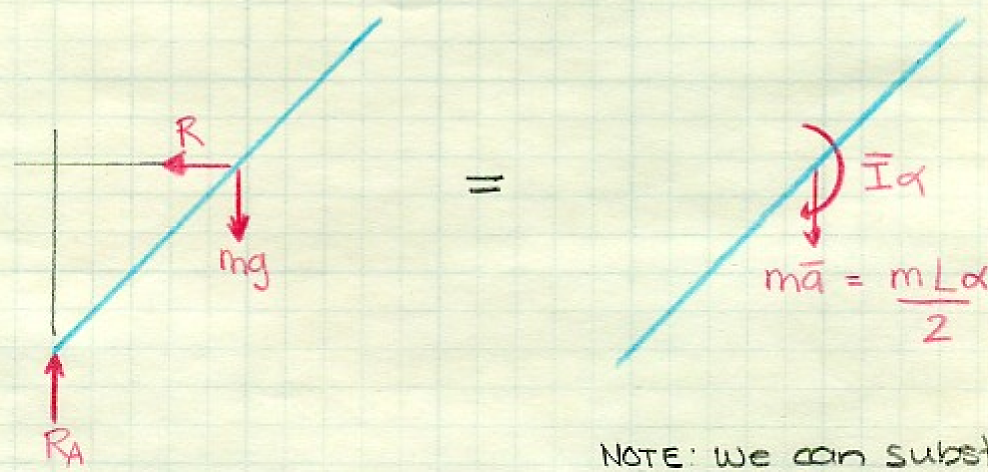
• Kinematic analysis:

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$
$$a_A \leftarrow\ = a_G \downarrow + a_{A/G} \ 30^{\circ}$$



$$a_G = a_{A/G} \sin 30^{\circ}$$
$$a_G = \frac{L\alpha}{2}$$

• Kinetics:



NOTE: we can substitute $\bar{a}$ because we know (or think we know) that $\bar{a}$ and α are related. (Since motion is constrained.)

Set #20

3. continued

$$\Sigma M_P = \Sigma M_{P_{eff}}$$

$$mgL\cos 60° = \frac{mL\alpha}{2}(L\cos 60°) + \frac{1}{2}\frac{W}{g}(2L)^2\alpha$$

$$mgL(.5) = \frac{mL\alpha}{2}(L)(.5) + \frac{1}{12}\frac{W}{g}(4L^2)\alpha$$

$$\frac{W}{2} = \frac{WL\alpha}{2g\,2} + \frac{1}{3}\frac{W}{g}L\alpha$$

$$6g = 3L\alpha + 4L\alpha$$

$$6g = 7L\alpha$$

$$\rightarrow \alpha = \frac{6g}{7L}$$

$$\underline{\underline{\alpha = 22.08 \ \text{rad}/s^2}}$$

(b.) $\Sigma F_x = \Sigma F_{x\,eff}$

$$-R = 0$$

$$R = 0$$

$$\Sigma F_y = \Sigma F_{y\,eff}$$

$$-14 + R_A = -\frac{mL\alpha}{2}$$

$$R_A = -\left(\frac{14 \text{ lbs}}{32.2 \text{ ft}/s^2}\right)\left(\frac{1.25 \text{ ft}}{2}\right)\left(22.08 \ \text{rad}/s^2\right) + 14 \text{ lb}$$

$$\underline{\underline{R_A = 8 \text{ lbs}}}$$