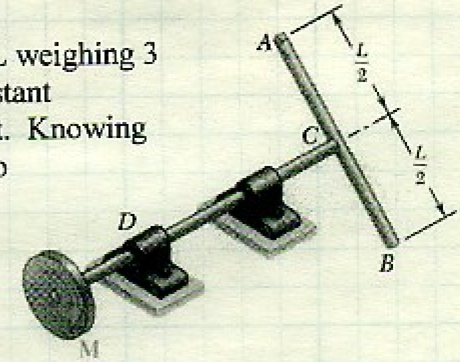
Set #21 – Work and Energy for Rigid Bodies



1. A 9-inch-diameter disk weighing 8 lb and rod AB of length L weighing 3 lb/ft are attached to the shaft CD as shown. A couple **M** of constant magnitude 4ft-lb is applied to the disk when the system is at rest. Knowing that the angular velocity of the system is to be 300 rpm after two complete revolutions, determine the required length L of the rod.

Given: $r_{disk} = 4.5 \text{ in} = .375 \text{ ft}$

$W_{disk} = 8 \text{ lb}$

$\frac{W}{L}_{rod} = 3 \text{ lb/ft}$

$v_0 = \omega_0 = 0$

$\vec{M} = 4 \text{ ft}\cdot\text{lb}$

$\omega_f = 300 \text{ rpm} = 31.416 \text{ rad/s}$ after 2 rev. (4π rad)

Find: L

Solution: Use $T_1 + U_{1\to2} = T_2$

$T_1 = 0$

$$U_{1\to2} = M\theta = (4 \text{ ft}\cdot\text{lb})(4\pi \text{ rad}) = 50.265 \text{ ft}\cdot\text{lbs.}$$

$$T_2 = \tfrac{1}{2}\bar{I}_{disk}\,\omega^2 + \tfrac{1}{2}\bar{I}_{rod}\,\omega^2$$

$$\Rightarrow \bar{I}_{disk} = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{8 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(.375 \text{ ft}) = 0.01747 \text{ ft}\cdot\text{lb}\cdot\text{s}^2$$

$$\bar{I}_{rod} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{\left(\frac{3 \text{ lb}}{\text{ft}}L\right)}{32.2 \text{ ft/s}^2}\right)L^2 = .007764\,L^3$$

$$T_2 = \tfrac{1}{2}(0.0175 \text{ ft}\cdot\text{lb}\cdot\text{s}^2)(31.416 \text{ rad/s})^2 + \tfrac{1}{2}(.007764L^3)(31.416 \text{ rad/s})^2 = 8.636 + 3.83L^3$$

$$T_1 + U_{1\to2} = T_2$$
$$0 + 50.265 = 8.636 + 3.83L^3$$

$$\rightarrow \underline{\underline{L = 2.2 \text{ ft}}}$$

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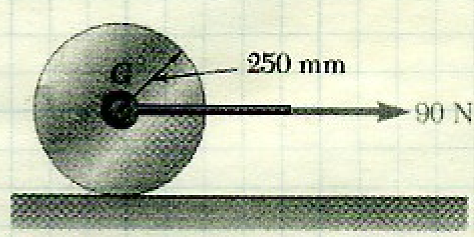
2. A 20-kg uniform cylindrical roller, initially at rest, is acted upon by a 90-N force as shown. Knowing that the body rolls without slipping, determine

a) the velocity of its center G after it has moved 1.5 m,

b) the friction force required to prevent slipping.



Given: $m = 20 \text{ kg}$

$V_o = \omega_o = 0$

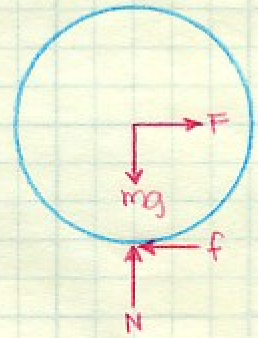
$F = 90 \text{ N}$

No slip

Find: (a.) $\bar{v}$ after 1.5 m

(b.) f for no slip

Solution: $T_1 + U_{1\to 2} = T_2$



$T_1 = 0$

$$U_{1\to 2} = F \cdot d = (90 \text{ N})(1.5 \text{ m}) = 135 \text{ N}\cdot\text{m}$$

$$T_2 = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2$$

$$\to \bar{I}_{cyl} = \tfrac{1}{2} m r^2 = \tfrac{1}{2}(20 \text{ kg})(.25 \text{ m})^2 = .625 \text{ kg}\cdot\text{m}^2$$

$$T_2 = \tfrac{1}{2}(20 \text{ kg})(\omega r)^2 + \tfrac{1}{2}(.625 \text{ kg}\cdot\text{m}^2)\omega^2$$
$$= \omega^2\left(\tfrac{1}{2}(20\text{kg})(.25\text{m})^2 + \tfrac{1}{2}(.625 \text{ kg}\cdot\text{m}^2)\right)$$
$$= (.9375 \text{ kg}\cdot\text{m}^2)\omega^2$$

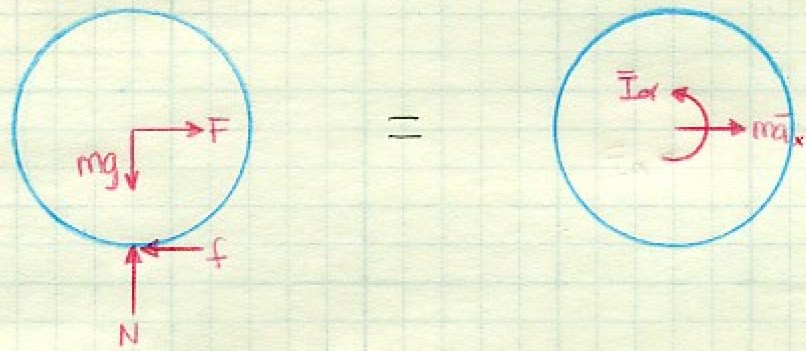
$$T_1 + U_{1\to 2} = T_2$$
$$0 + 135 \text{ N}\cdot\text{m} = (.9375 \text{ kg}\cdot\text{m}^2)\omega^2$$

$$\longrightarrow \omega = 12 \text{ rad/s}$$

$$\bar{v} = \omega r$$
$$= (12 \text{ rad/s})(.25\text{m})$$
$$\underline{\bar{v} = 3 \text{ m/s} \longrightarrow}$$

Set # 21

2. continued



$\Sigma F_x = ma_x$

$F - f = ma_x$

$90N - f = ma_x$ — 1

$\Sigma M_G = I\alpha$

$\left(\text{NOTE: } a_x = \alpha r \quad \rightarrow \alpha = \dfrac{a_x}{r} \right)$

$fr = \left(\frac{1}{2} m\cancel{r^2}\right)\left(\frac{a_x}{\cancel{r}}\right)$

$f = \frac{1}{2}(20)\, a_x$

$f = 10\, kg\ a_x$

$\rightarrow a_x = 0.1 f$ — 2

2 in 1

$90N - f = m\,(0.1 f)$

$90N - f = (20\, kg)(0.1\, f)$

$90N - f = 2f$

$\rightarrow \underline{\underline{f = 30N}} \leftarrow$

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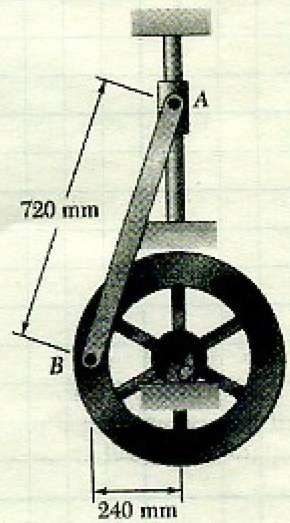
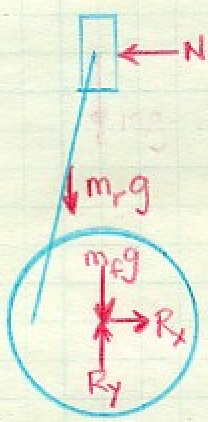
3. The 4-kg rod AB is attached to a collar of negligible mass at A and to a flywheel at B. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when point B is directly below C.



Given: $m_r = 4\text{ kg}$
$m_f = 16\text{ kg}$
$K = 180\text{ mm}$
$\omega_o = 60\text{ rpm} = 6.2832\text{ rad/s}$ ↻
Frictionless

Find: ω_f when B is directly below C ($= \omega$)

Solution:



The only forces that do work are $m_r g$ and $m_f g$. However, $m_f g$ does not because the flywheel does not move.

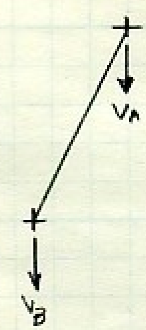
Use Work-Energy Theorem

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \tfrac{1}{2} m \bar{v}_r^2 + \tfrac{1}{2} \bar{I}_r \omega_r^2 + \tfrac{1}{2} \bar{I}_f \omega_f^2$$

$$\rightarrow \bar{I}_f = m_f k_f^2$$
$$= (16\text{ kg})(.18\text{ m})^2$$
$$= .5184\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_r = \tfrac{1}{12} m \ell^2$$
$$= \tfrac{1}{12}(4\text{ kg})(.72\text{ m})^2$$
$$= .1728\text{ kg}\cdot\text{m}^2$$

To find $\bar{v}_r$: Rod is in translation $\rightarrow \omega_r = 0$



$$\vec{v}_r = \vec{v}_B = \omega_f r_f$$
$$= (6.2832\text{ rad/s})(.24\text{ m})$$
$$= 1.507944\text{ m/s}$$

Set #21

3. continued

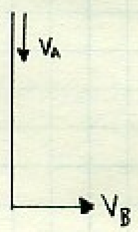
$$T_1 = \tfrac{1}{2}(4\,kg)(1.508\,m/s)^2 + 0 + \tfrac{1}{2}(.5184\,kg \cdot m^2)(6.28\,rad/s)^2$$
$$= 14.77\,J$$

$$V_1 = mgh_1$$
$$= (4\,kg)(9.81\,m/s^2)(.3415\,m)$$
$$= 13.40\,J$$

$$V_2 = mgh_2$$
$$= (4\,kg)(9.81\,m/s^2)(.12\,m)$$
$$= 4.7088\,J$$

$$T_2 = \tfrac{1}{2} m \bar{v}_r^2 + \tfrac{1}{2} \bar{I}_r \omega_r^2 + \tfrac{1}{2} \bar{I}_f \omega_f^2$$

To find $\bar{v}_r$:



A is instant center:

$$V_B = \omega_r l = \omega_f r$$
$$\longrightarrow \omega_r = \frac{r}{l}\omega_f$$
$$= \frac{\omega_f}{3}$$

$$\bar{v}_r = \omega_r \frac{l}{2}$$
$$= \left(\frac{\omega_f}{3}\right)\frac{l}{2}$$
$$= .12\,\omega_f$$

$$T_2 = \tfrac{1}{2}(4\,kg)\left[\frac{\omega_f}{3}\,\frac{l}{2}\right]^2 + \tfrac{1}{2}(.1728\,kg \cdot m^2)\left(\frac{\omega_f}{3}\right)^2$$
$$+ \tfrac{1}{2}(.5184\,kg \cdot m^2)\,\omega_f^2$$

$$= \omega_f^2\,[(\tfrac{1}{2})(4)(\tfrac{1}{9})(\tfrac{1}{4})(.72)^2 + (\tfrac{1}{2})(.1728)(\tfrac{1}{9}) + (\tfrac{1}{2})(.5184)]$$
$$= \omega_f^2\,(.0288 + .0096 + .2592$$
$$= .2976\,\omega_f^2$$

$$T_1 + V_1 = T_2 + V_2$$
$$14.77\,J + 13.40\,J = .2976\,\omega_f^2 + 4.7088\,J$$

$$\longrightarrow \omega_f = 8.879\,rad/s$$
$$\underline{\underline{\omega_f = 84.79\,rpm}}$$