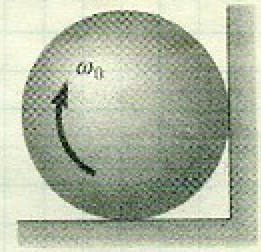
Set #22 – Impulse and Momentum for Rigid Bodies

1. A 3-kg sphere of radius r = 125 mm with an initial clockwise angular velocity ω_0 = 90 rad/s is placed in the corner formed by the floor and a vertical wall. Knowing that the coefficient of kinetic friction is 0.10 at A and B, determine the time required for the sphere to come to rest.



Given: $m = 3\text{kg}$

$r = 125\text{ mm} = .125\text{ m}$

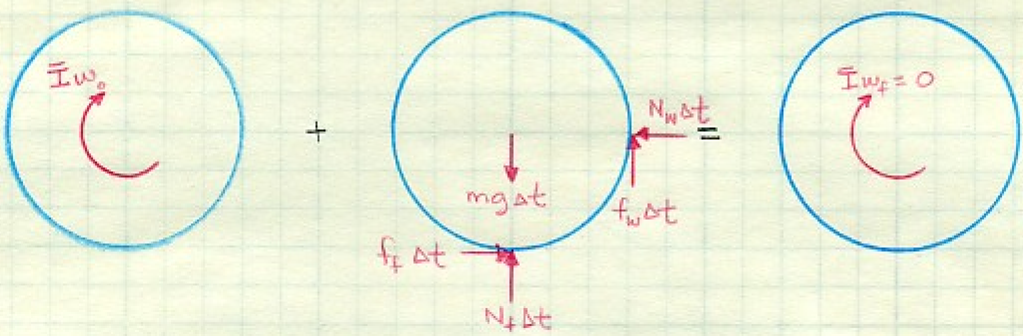
$\omega_0 = 90 \text{ rad/s}$ ↻

$\mu_k = 0.10$

Find: t to come to rest

Solution:

$\vec{L}_1 + \vec{I}_{1\to2} = \vec{L}_2$



x: $\mu_k N_f \Delta t - N_w \Delta t = 0$

$\mu_k N_f = N_w$ — ①

y: $N_f \Delta t - mg \Delta t + \mu_k N_w \Delta t = 0$

$N_f - mg + \mu_k N_w$

$N_f = mg - \mu_k N_w$ — ②

Substitute ① into ②

$N_f = mg - \mu_k (\mu_k N_f)$

$N_f = mg - \mu_k^2 N_f$

$N_f(1 + \mu_k^2) = mg$

$N_f = 29.14\text{ N}$ ↑

$N_w = 2.914\text{ N}$ ←

Set #22

1. continued

$\circlearrowleft^{+}_{G}$:

$$-\bar{I}\omega_o + (\mu_k N_f \Delta t) r + (\mu_k N_w \Delta t) r = 0$$

$$-\left(\tfrac{2}{5} m r^2\right)\omega_o + (\mu_k N_f \Delta t) r + (\mu_k N_w \Delta t) r = 0$$

$$\Delta t (\mu_k N_f r + \mu_k N_w r) = \tfrac{2}{5} m r^2 \omega_o$$

$$\Delta t = \frac{2}{5} \frac{m r^2 \omega_o}{\mu_k N_f r + \mu_k N_w r}$$

$$= \frac{2}{5} \frac{(3)(.125)^2(90)}{(.1)(29.14)(.125) + (.1)(2.914)(.125)}$$

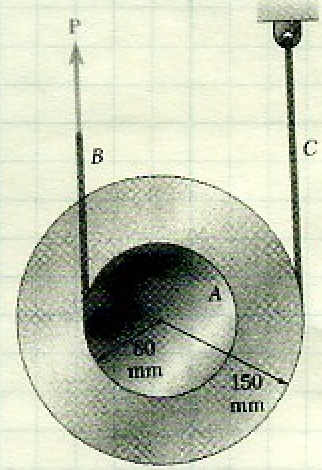
$$\underline{\underline{\Delta t = 4.21 \text{ sec}}}$$

Set #22 – Impulse and Momentum for Rigid Bodies

2. The double pulley shown has a mass of 3 kg and a radius of gyration of 100mm. Knowing that when the pulley is at rest, a force **P** of magnitude 24 N is applied to cord B, determine

a) the velocity of the center of the pulley after 1.5 s,

b) the tension in cord C.



Given: $m = 3\text{kg}$

$\bar{k} = 100 \text{ mm} = .100 \text{ m}$

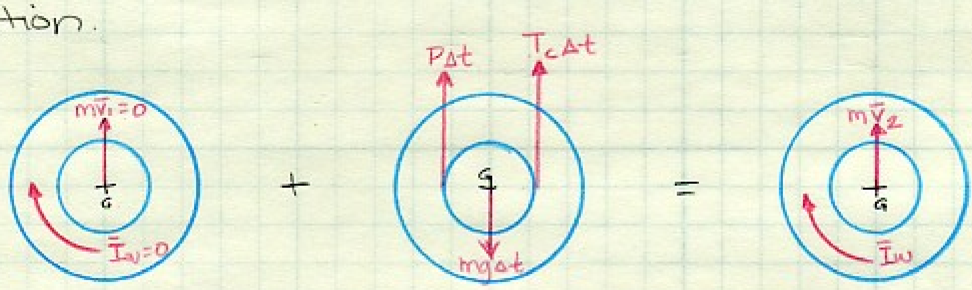
$v_0 = \omega_0 = 0$

$P = 24 \text{ N}$ applied

Find: (a) v_G after 1.5 sec.

(b) T_c

Solution.



$y: \quad P\Delta t + T_c\Delta t - mg\Delta t = m\bar{v}_2$

$P\Delta t + T_c\Delta t - mg\Delta t = m\,\omega_2\, r$

$(24\text{N})(1.5\text{s}) + T_c(1.5\text{s}) - (3\text{kg})(9.81\,\text{m/s}^2)(1.5\text{s}) = (3\text{kg})(\omega_2)(.15\text{m})$

$36\text{ N}\cdot\text{s} + 1.5\,T_c - 44.15\text{ N}\cdot\text{s} = .45\,\omega_2$

$\rightarrow \omega_2 = \dfrac{36\text{ N}\cdot\text{s} + 1.5\,T_c - 44.15\text{ N}\cdot\text{s}}{.45\text{ kg}\cdot\text{m}}$

$\omega_2 = 80 + 3.33\,T_c - 98.11$ — (1)

$\circlearrowright_G: \quad P\Delta t(.08\text{m}) - T_c\Delta t(.15\text{m}) = \bar{I}\omega_2$

$P\Delta t(.08\text{m}) - T_c\Delta t(.15\text{m}) = m\bar{k}^2\omega_2$

$(24)(1.5)(.08) - T_c(1.5)(.15) = (3)(.1)^2\,\omega_2$

$2.88 - .225\,T_c = .03\,\omega_2$ — (2)

Set # 22

2. continued

Substitute ① into ②.

$$2.88 - .225\,T_c = .03\,(80 + 3.33\,T_c - 98.11)$$
$$2.88 - .225\,T_c = 2.4 + .0999\,T_c - 2.9433$$
$$.325\,T_c = 3.4233$$
$$\underline{\underline{T_c = 10.53\ \text{N} \uparrow}}$$

$$\underline{\underline{\omega_2 = 17.01\ \text{rad/s} \circlearrowleft}} \quad \text{(from ①)}$$

$$v = \omega r$$
$$= (17.01\ \text{rad/s})(.15\ \text{m})$$
$$\underline{\underline{= 2.55\ \text{m/s} \uparrow}}$$