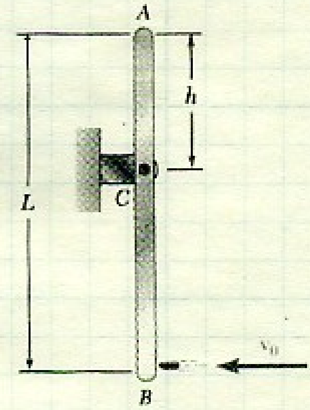
Set #23 – Eccentric Impact for Rigid Bodies

1. A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length L = 30 inches. Knowing that h = 12 inches and that the bar is initially at rest, determine

a) the angular velocity of the bar immediately after the bullet becomes embedded,

b) the impulsive reaction at C,

assuming that the bullet becomes embedded in 0.001 s.



Given: $W_b = 0.08$ lb

$V_{b_0} = 1800$ ft/s

$W_r = 15$ lb

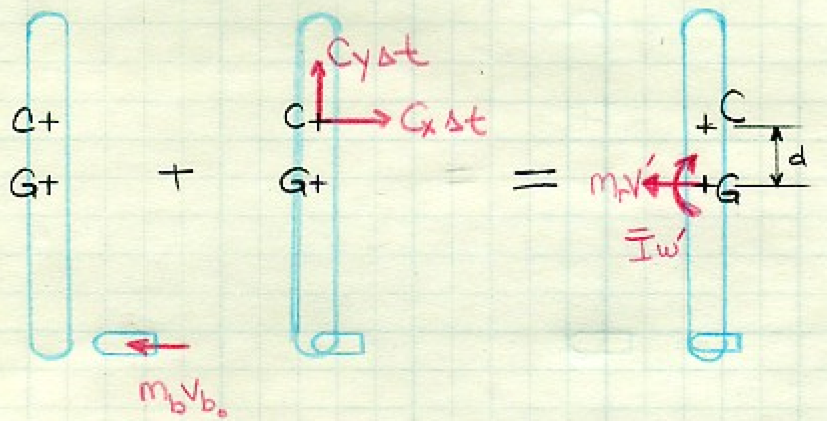
$L = 30$ in

$h = 12$ in

$V_{r_0} = 0$

Find: (a) ω'

(b) $\vec{C}$ assuming $\Delta t = 0.001$ s to become embedded



$$\Sigma X \xrightarrow{+} : \; -m_b v_0 + C_x \Delta t = -m_r v'$$

$$\longrightarrow C_x = \frac{m_b v_0 - m_r v'}{\Delta t}$$

$$= \frac{m_b v_0 - m_r d\omega'}{\Delta t} \quad \text{—(1)}$$

$$\Sigma y \uparrow + : \quad 0 + C_y \Delta t = 0$$

$$C_y = 0$$

Set # 23

1. continued

$$\Sigma M_c \circlearrowright: \quad m_b v_{b_0}\left(\frac{18\text{ in}}{12\text{ in}}\right)' + 0 = m_r v'\left(\frac{3\text{ in}}{12\text{ in}}\right)' + \bar{I}\omega'$$

$$\left(\frac{.08\text{ lb}}{32.2}\right)(1800\text{ ft/s})\left(\frac{18}{12}\text{ ft}\right) = \left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{3}{12}\text{ ft}\right)\omega'\left(\frac{3}{12}\text{ ft}\right) + \bar{I}\omega'$$

$$6.708 = .02911\,\omega' + \left(\frac{1}{12}\right)\left(\frac{15}{32.2}\right)\left(\frac{30}{12}\right)^2\omega'$$

$$6.708 = (.02911 + .24262)\,\omega'$$

$$6.708 = .2717\,\omega'$$

$$\longrightarrow \underline{\underline{\omega' = 24.689 \text{ rad/s}}}$$

We can substitute this value into ①.

$$C_x = \frac{\left(\frac{.08\text{ lb}}{32.2}\right)(1800\text{ ft/s}) - \left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{3}{12}\text{ ft}\right)(24.689\text{ rad/s})}{.001\text{ sec}}$$

$$\underline{\underline{C_x = 1597\text{ lbs}}}$$

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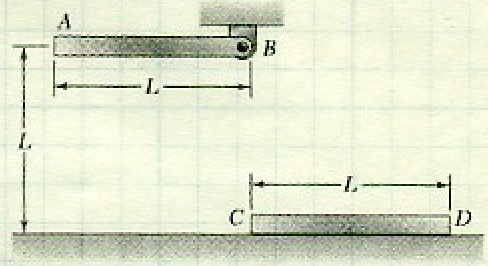
2. A slender rod AB is released from rest in the position shown. It swings down to a vertical position and strikes a second and identical rod CD which is resting on a frictionless surface. Assuming that the coefficient of restitution between the rods is 0.5, determine the velocity of rod CD immediately after the impact.



Given: $\omega_{AB_0} = 0$

$v_{CD_0} = 0$

$e = 0.5$

Find: $v_{CD}' = v_C'$

Solution:

1. We must first v_A just before its impact with C.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgL = \tfrac{1}{2}\bar{I}\omega^2 + \tfrac{1}{2}m\bar{v}_{AB}^2 + mg\frac{L}{2} \qquad (\text{NOTE: } \bar{v}_{AB} = \tfrac{1}{2}L\omega)$$

$$mgL = \tfrac{1}{2}\bar{I}\omega^2 + \tfrac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + mg\frac{L}{2}$$

$$\cancel{m}g\frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{12}\cancel{m}L^2\right)\omega^2 + \frac{1}{\cancel{2}}\cancel{m}\left(\frac{L}{2}\omega\right)^2$$

$$gL = \tfrac{1}{12}L^2\omega^2 + \tfrac{1}{4}L^2\omega^2$$

$$g = \left(\tfrac{1}{12}L - \tfrac{1}{4}L\right)\omega^2$$

$$g = \tfrac{4}{12}L\omega^2$$

$$g = \tfrac{1}{3}L\omega^2$$

$$\longrightarrow \omega = \sqrt{\frac{3g}{L}}$$

$$\bar{v}_{AB} = \sqrt{\frac{3g}{L}}\left(\frac{L}{2}\right)$$

$$v_A = \omega L$$

$$= L\sqrt{\frac{3g}{L}}$$

Set #23

2. continued

2 Impact:

$$V'_{Cn} - V'_{An} = e\,(V_{An} - \cancel{V_{Cn}}^{\,0})$$

$$V'_C - V'_A = eL\sqrt{\frac{3g}{L}}$$

$$\longrightarrow V'_C = V'_A + eL\sqrt{\frac{3g}{L}}$$

$$V'_C = \omega' L + eL\sqrt{\frac{3g}{L}}$$

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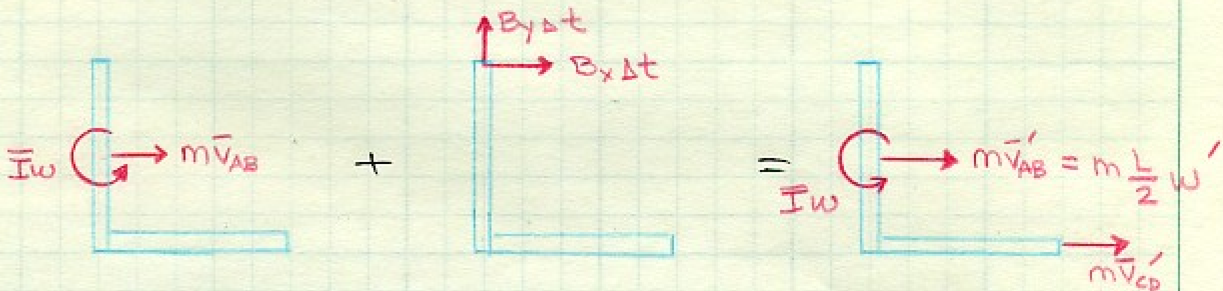

$$\omega = \sqrt{\frac{3g}{L}}$$
$$\bar{I} = \tfrac{1}{12} mL^2$$
$$\bar{v}_{AB} = \tfrac{1}{2} L\omega = \tfrac{1}{2} L\sqrt{\frac{3g}{L}}$$

$$\omega'_{CD} = 0$$
$$\bar{I} = \tfrac{1}{12} mL^2$$
$$v'_{CD} = V'_C \quad \text{(trans. only)}$$
$$V'_C = \omega' L + eL\sqrt{\frac{3g}{L}}$$

$$\Sigma M_B \circlearrowleft: \quad \bar{I}\omega + m\bar{v}_{AB}\frac{L}{2} + 0 = \bar{I}\omega' + m\bar{v}'_{AB}\frac{L}{2} + m\bar{v}'_{CD}L$$

$$\frac{mL^2}{12}\sqrt{\frac{3g}{L}} + \frac{mL^2}{4}\sqrt{\frac{3g}{L}} = \frac{mL^2}{12}\omega' + \frac{mL^2}{4}\omega' + mLV'_C$$

$$\frac{mL^2}{3}\sqrt{\frac{3g}{L}} = \frac{mL^2}{3}\omega' + mLV'_C$$

$$\longrightarrow \omega' = \left(\frac{mL^2}{3}\sqrt{\frac{3g}{L}} - mLV'_C\right)\frac{3}{mL^2}$$

$$\omega' = \sqrt{\frac{3g}{L}} - \frac{3V'_C}{L}$$

Set #23

2. continued

Substitute the latter into part 2.

$$v_c' = \left(\sqrt{\frac{3g}{L}} - \frac{3v_c'}{L}\right)L + eL\sqrt{\frac{3g}{L}}$$

$$= L\sqrt{\frac{3g}{L}} - 3v_c' + eL\sqrt{\frac{3g}{L}}$$

$$4v_c' = (1+e)\,L\sqrt{\frac{3g}{L}}$$

$$v_c' = \frac{1+e}{4}\sqrt{3gL}$$

$$v_c' = .375\sqrt{3gL} \longrightarrow$$

$$\omega' = \sqrt{\frac{3g}{L}} - \frac{3(.375\sqrt{3gL})}{L}$$

$$\omega' = \sqrt{\frac{3g}{L}} - 3(.375)\sqrt{\frac{3g}{L}}$$

$$\omega' = [1 - 3(.375)]\sqrt{\frac{3g}{L}}$$

$$\omega' = -0.125\sqrt{\frac{3g}{L}}$$

$$\omega' = -\frac{1}{8}\sqrt{\frac{3g}{L}}$$

$$\omega' = \frac{1}{8}\sqrt{\frac{3g}{L}}$$